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Constrained Model Predictive Control for Time-varying Delay Systems: Application to an Active Car Suspension

Sofiane Bououden, Mohammed Chadli *, Lixian Zhang, and Ting Yang

Abstract: This study investigates the problem of robust model predictive control (RMPC) for active suspension systems with time-varying delays and input constraints. The uncertainty is of convex polytopic type. Based on the Lyapunov-Krasovskii functional method, sufficient stability conditions of the time-varying delays systems are derived by linear matrix inequalities (LMIs) terms. At each time set, a feasible state feedback is obtained by minimizing an upper bound of the 'worst-case' quadratic objective function over an infinite horizon subject to constraints on inputs. Finally, a quarter-vehicle model is exploited to demonstrate the effectiveness of the proposed method.

Keywords: Active suspension systems, input constraints, LMIs constraints, model predictive control, structured uncertainty, time-varying delays.

1. INTRODUCTION

Time delay often occurs in many dynamical systems, such as chemical processes, communication systems, and vehicle systems [1–3]. Presence of time delay in a process increases the difficulty of controlling it. These delays can affect the state input or/and output, and they can be constant or time varying, known or unknown, deterministic or stochastic depending, on the systems under consideration [5, 6].

Recently, many researchers focuses on the robust model predictive control (RMPC) [4, 7, 18, 19, 22, 23]. For polytopic uncertainty or structured feedback uncertainty with fixed or time-varying state and/or input delays [20, 21], the design of MPC controller has been proposed in [8, 9]. On the other hand, in [11, 12] an algorithm for polytopic uncertain systems with state delay is presented, and [12] proposed a robust model RMPC with constant state delay using linear matrix inequalities (LMIs) terms. The work of [13] proposes an MPC algorithm for uncertain time-varying systems with state-delays. Indeed, [13] and [14] considered time-varying delays and presented an adaptive power and rate control algorithm, where the delay time is assumed to be unknown but with a known upper bound. However, at the best of our knowledge, these algorithms did not consider input constraints and uncertainties combined with time-varying delays.

In this paper, we propose a RMPC algorithm for a class of uncertain discrete-time system with time-varying delays and input constraints. The uncertainty is assumed to be polytopic with a known upper bound. A Lyapunov-Krasovskii function is constructed for deriving an upper bound of the cost function, and then by minimizing the upper bound, an MPC state feedback law is obtained in terms of LMIs. Finally, the constrained control problem for an uncertain quarter-vehicle model subject to time-varying delays is studied using the proposed algorithm.

The rest of the paper is organized as follows. In Section 2, the problem formulation for constrained MPC of structured uncertain systems subject to time-varying delay is introduced. Section 3 presents the main results of this paper, in which the robust MPC approach is derived using LMIs formulation. Section 4 illustrates the performance of the proposed controller through an uncertain quarter-vehicle model subject to time-varying delays. Section 5 concludes this paper.

2. MODEL PREDICTIVE CONTROLLER DESIGN

Consider the following discrete-time system with time-varying delay:

$$\begin{cases} x(k+1) = A(k)x(k) + A_d(k)x(k-d_k) \\ \qquad\qquad + B(k)u(k-d_k), \\ y(k) = Cx(k). \end{cases} \tag{1}$$

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Sofiane Bououden is with the Faculty of sciences and technology, University of Abbes Laghrour Khenchela, Algeria (e-mail: ss.bououden@gmail.com). Mohammed Chadli is with the University of Picardie Jules Verne, MIS (EA 4029), 33 rue Saint-Leu, 80039 Amiens, France (e-mail: mchadli@u-picardie.fr). Lixian Zhang and Ting Yang are with the Harbin Institute of Technology, Harbin, 150080 China (e-mails: lixianzhang@hit.edu.cn, tingyang@hit.edu.cn).
* Corresponding author.

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The initial condition is $x(k) = \phi(k)$, $k = [-d_M, 0]$, $x(k) \in \Re^n$ is the state system vector, $u(k) \in \Re^m$ is the control input, d_k is a time-varying delay and satisfies $0 \le d_m \le d_k \le d_M$, where d_m and d_M are the lower bound and upper bound of d_k, respectively and $[A(k)\;\; A_d(k)\;\; B(k)] \in \Omega$. For polytopic uncertainty, the set Ω is the polytope

$$Co\{[A_1(k)\, A_{d1}(k)\, B_1(k)], \ldots, [A_r(k)\, A_{dr}(k)\, B_r(k)]\}$$

where Co devotes to the convex hull, with known vertices $A_i \in \Re^{n\times n}$, $A_{di} \in \Re^{n\times n}$, $B_i \in \Re^{m\times m}$, $i = 1, 2, \ldots, r$. Any $[A(k), A_d(k), B(k)]$ within the convex set Ω is a linear combination of the vertices

$$[A(k), A_d(k), B(k)] = \sum_{i=1}^{r} \delta_i [A_i A_{di}\, B_i] \tag{2}$$

with $\sum_{i=1}^{r} \delta_i = 1$, $0 \le \delta_i \le 1$.

Now we are ready to give a problem statement of the RMPC with a quadratic objective function subject to input constraints. Indeed, at each time, the control objective of the MPC problem is to compute the control move $u(k+i/k)$ by minimizing the following performance index:

$$\min_{u(k+i/k),\, i\ge 0} \max J_\infty(k) = \sum_{i=0}^{\infty} \|x(k+i/k)\|_Q^2 + \|u(k+i/k)\|_R^2 \tag{3}$$

$$|u_j(k+i/k)| \le u_{j,\max}\,,\; i \ge 0,\;\; j = 1, \ldots, q, \tag{4}$$

where $Q > 0$, $R > 0$ are two known weighting matrices.

Equations (3) and (4) are a constrained min-max optimization problem corresponding to a worst-case infinite-horizon MPC with a quadratic objective, which is not tractable in general, and relaxed to another minimization problem.

Before ending this section, we introduce the following lemmas, which will play an important role in the next section.

Lemma 1: There exist a matrix $P > 0$ and a symmetric matrix Q such that the following conditions are equivalent:

1) $A^T PA - Q < 0$

2) $\begin{bmatrix} -Q & A^T P \\ PA & -P \end{bmatrix} < 0.$

Lemma 2: For any matrix P, integers a_1 and a_2 satisfying $a_1 < a_2$, and vector function $x(v) : [k-a_2, k-a_1] \to \Re^n$, the following equality holds:

$$x^T(k-a_1)Nx(k-a_1) - x^T(k-a_2)Nx(k-a_2) = \sum_{v=k-a_2}^{k-a_1-1} \left(x^T(v+1)Nx(v+1) - x^T(v)Nx(v)\right). \tag{5}$$

Lemma 3: For any positive integer c, and matrices W and H with appropriate dimensions, the following inequality holds:

$$W^T HW \le -2cW - c^2 H^{-1}. \tag{6}$$

3. MAIN RESULTS

In this section, we propose a state feedback control approach for uncertain time-varying delay systems.

Lemma 4 [13, 16]**:** If there exist variables $\gamma > 0$, $X > 0$, $W > 0$ satisfying the following conditions for all $i = 1, 2, \ldots, r$:

$$\min_{\gamma, Q, W, Y} \gamma \tag{7}$$

subject to

$$\begin{bmatrix} 1 & x^T(k) & x^T(k-1) & \cdots & x^T(k-d) \\ * & X & 0 & \cdots & 0 \\ * & * & W & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ * & * & * & \cdots & W \end{bmatrix} > 0, \tag{8}$$

$$\begin{bmatrix} u_{\max}^2 & Y \\ * & X \end{bmatrix} > 0, \tag{9}$$

$$\begin{bmatrix} X & Y^T R^{1/2} & XX^{1/2} & X & \mathbb{X} & 0 \\ * & \gamma I & 0 & 0 & 0 & 0 \\ * & * & \gamma I & 0 & 0 & 0 \\ * & * & * & W & 0 & 0 \\ * & * & * & * & X & A_{d,i} \\ * & * & * & * & * & W^{-1} \end{bmatrix} > 0, \tag{10}$$

where $\mathbb{X} \triangleq XA^T + Y^T B^T$. Then the control law $u(k+j/k) = Fx(k+j/k)$ with $F = YX^{-1}$ guarantees the stability of the system (1) with constant delay $(d_k = d)$, constraints on input (3) and (4).

Remark 1: The above result assumed known and constant time delay [13, 16]. Though widely accepted, this assumption is still restrictive because many systems in practice do not satisfy this assumption.

Let us now consider the following state-feedback control law

$$u(k+j/k) = Fx(k+j/k)$$

with $x(k+j/k) = x(k/k)$ for $j \ge 0$.

Then, the closed-loop system can be written as:

$$x(k+1) = Ax(k) + (A_d + BF)x(k-d_k). \tag{11}$$

For the sake of simplicity on matrix representation, we define $\alpha_i \in \Re^{4n\times n}$ $(i = 1, \ldots, 4)$ as follows:

$$\alpha_1 = [I_n, 0_n, 0_n, 0_n], \alpha_2 = [0_n, I_n, 0_n, 0_n],$$
$$\alpha_3 = [0_n, 0_n, I_n, 0_n], \alpha_4 = [0_n, 0_n, 0_n, I_n].$$

In the following, we introduce two vectors:

$$\xi(k) = \left[x^T(k)\, x^T(k-d_m)\right]^T,$$
$$\delta(k) = \left[\xi^T(k)\, \xi^T(k-d_k)\, \xi^T(k-d_M)\, \xi^T(k-d_m)\right]^T$$

then,

$$x(k+1) = \mathrm{E}_3\xi(k), x(k) = \mathrm{E}_1\xi(k),$$
$$x(k-d_k) = \mathrm{E}_2\xi(k), \xi(k) = \alpha_1\delta(k)$$

with

$$\mathrm{E}_1 = [I\ 0]$$
$$\mathrm{E}_2 = [0\ I]$$
$$\mathrm{E}_1 = [A\ \ (A_d + BF)]$$

where 0 and I denote a matrix of zeros and on identity matrix, respectively, with appropriate dimensions.

Following similar approach in [16], to find an upper bound of the cost function $J_\infty(k)$, a Lyapunov-Krasovskii functional candidate, at time k, is considered as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k) \tag{12}$$

where

$$V_1(k) = x^T(k)Sx(k)$$
$$V_2(k) = \sum_{v=k-d_k}^{k-1} \xi(v)N\xi(v)$$
$$V_3(k) = \sum_{h=-d_M+1}^{-d_m+1} \sum_{s=k-1+h}^{k-1} \xi^T(s)M\xi(s)$$

with S, N, M are symmetric positive matrices.

An upper bound on the worst value of the cost function $J(k)$ is obtained when the following stability constraint is satisfied for any $[A(k+j+1/k)\ \ A_d(k+j+1/k)] \in \Omega,\ \ j \geq 0$

$$\begin{aligned} &V(x(k+j+1/k)) - V(x(k+j/k)) \\ &\quad \leq -\left[\|\bar{x}(k+j/k)\|_Q^2 + \|u(k+j/k)\|_R^2\right]. \end{aligned} \tag{13}$$

As it is assumed that the summation is up to ∞, ie., $j \to \infty$, $x(\infty) = 0$. Summing (13) from $j = 0$ to ∞ yields $J(k) \leq V(x(k/k))$.

By defining $V(x(k/k)) \leq \gamma$, an upper bound on the cost function is obtained as $J(k) \leq \gamma$.

The main result in this paper is presented in the following theorem.

Theorem 1: Consider the closed-loop time-varying delay system (11) at each time k and let $x(k/k)$ be the measured state $x(k)$. There exists a state feedback control law, meeting the performance objective (3) for $d_m \leq d_k \leq d_M$, if there exist symmetric positive definite matrices $X > 0$, Y, G and W satisfying the following convex optimization problem:

$$\min_{g,X,Y,G,W} g \tag{14}$$

subject to

$$\begin{bmatrix} -1 & x^T(k/k) \\ x(k/k) & -X \end{bmatrix} < 0 \tag{15}$$

$$\begin{bmatrix} -u_{\max}^2 & Y^T \\ Y & -X \end{bmatrix} < 0 \tag{16}$$

$$\begin{bmatrix} \Xi & (\Gamma X + \Theta Y)^T & \Lambda^T X^T & \Phi^T X^T \\ * & -X & 0 & 0 \\ * & * & -X & 0 \\ * & * & * & X \end{bmatrix} < 0 \tag{17}$$

$$\begin{bmatrix} -2(d+1)\alpha_1^T\mu X & (d+1)\alpha_1^T\mu^2 & \alpha_1^T X^T & \alpha_2^T X^T & \alpha_3^T X^T & \alpha_4^T X^T & \alpha_1^T X^T Q^{\frac{1}{2}} E_3^T & \alpha_1^T E_2^T Y^T R^{\frac{1}{2}} \\ * & -W^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -G & 0 & 0 & 0 & 0 & 0 \\ * & * & * & G & 0 & 0 & 0 & 0 \\ * & * & * & * & W & 0 & 0 & 0 \\ * & * & * & * & * & W & 0 & 0 \\ * & * & * & * & * & * & gI & 0 \\ * & * & * & * & * & * & * & gI \end{bmatrix} < 0, \tag{18}$$

where $\Xi = X\Gamma^T\Lambda + Y^T\Theta^T\Lambda + \Lambda^T\Gamma X + \Lambda^T\Theta Y$, $\Lambda^T = \alpha_1^T E_1^T A^T$, $\Gamma^T = \alpha_1^T E_2^T A_d^T$, $\Theta^T = \alpha_1^T E_2^T B^T$, $\Phi^T = \alpha_1^T E_1^T$, $d = d_M - d_m$.

Then the state feedback controller $u(k) = Fx(k)$ minimizes the upper bound of the cost function, where the feedback matrix F is given by $F = YX^{-1}$.

Proof: Define the forward difference of $V(k)$ as $\Delta V(k) = V(k+1) - V(k)$. From the definition of quadratic function $V(x(k/k))$, it follows

$$\begin{aligned} &V(x(k+j+1/k)) - V(x(k+j/k)) \\ &\quad \leq -\left[\|x(k+j/k)\|_Q^2 + \|u(k+j/K)\|_R^2\right]. \end{aligned} \tag{19}$$

Using the state-feedback control law $u(k+j) = YX^{-1}x(k+j)$. According to system (11) we have

$$\begin{aligned} &V(k+1) - V(k) \\ &\quad < \delta(k)^T\alpha^T\left(E_3^T Q E_3 + E_2^T X^{-1T} Y^T R Y X^{-1} E_2\right)\alpha\delta(k) \end{aligned} \tag{20}$$

The difference of $V_1(k)$ with respect to time along the trajectory of system (1) is

$$\begin{aligned} \Delta V_1(k) &= x(k+1)^T Sx(k+1) - x(k)^T Sx(k) \\ &= \xi(k)^T\left(E_1^T A^T SAE_1 + E_2^T\left(A_d + BYX^{-1}\right)^T SAE_1\right. \\ &\quad \left.+ E_1 AS\left(A_d + BYX^{-1}\right)E_2\right)\xi(k) + \xi(k)^T\left(E_2^T\right. \\ &\quad \left.\times\left(A_d + BYX^{-1}\right)^T S\left(A_d + BYX^{-1}\right)E_2\right)\xi(k) \\ &\quad - \xi(k)^T E_1^T SE_1\xi(k) \end{aligned}$$

$$= \delta(k)^T \alpha_1^T \Big(E_1^T A^T SAE_1 + 2\Big(E_2^T \left(A_d + BYX^{-1}\right)^T \times SAE_1\Big) + E_1 AS\left(A_d + BYX^{-1}\right) E_2 + E_2^T \big(A_d + BYX^{-1}\big)^T S\left(A_d + BYX^{-1}\right) E_2 - E_1^T SE_1 \Big) \alpha_1 \delta(k). \tag{21}$$

The difference of $V_2(k)$ with respect to time along the trajectory of system (1) is

$$\Delta V_2(k) = \sum_{v=k+1-d_{k+1}}^{k} \xi(v)^T N \xi(v) - \sum_{v=k-d_k}^{k-1} \xi(v)^T N \xi(v).$$

By utilizing (5) of Lemma 2, the above matrix equation becomes:

$$\begin{aligned} \Delta V_2(k) &= \xi(k)^T N \xi(k) - \xi(k-d_k)^T N \xi(k-d_k) \\ &= \delta^T(k) \left(\alpha_1^T N \alpha_1 - \alpha_2^T N \alpha_2\right) \delta(k). \end{aligned} \tag{22}$$

The difference of $V_3(k)$ with respect to time along the trajectory of system (1) is

$$\begin{aligned} \Delta V_3(k) =& \sum_{h=-d_M+1}^{-d_m+1} \sum_{s=k+h}^{k} \xi^T(s) M \xi(s) \\ &- \sum_{h=-d_M+1}^{-d_m+1} \sum_{s=k-1+h}^{k-1} \xi^T(s) M \xi(s) \\ &= (d_M - d_m + 1)\xi^T(k) M \xi(k) - \sum_{s=k-d_M}^{k-d_m} \xi^T(s) M \xi(s) \\ &\le (d_M - d_m + 1)\xi^T(k) M \xi(k) \\ &- \xi^T(k-d_M) M \xi(k-d_M) - \xi^T(k-d_m) M \xi(k-d_m) \\ &= \delta^T(k)\big((d+1)\alpha_1^T M \alpha_1 - \alpha_3^T M \alpha_3 - \alpha_4^T M \alpha_4\big)\delta(k). \end{aligned} \tag{23}$$

Combining (21), (22), and (23), one obtains

$$\begin{aligned} &\delta(k)^T \alpha_1^T \Big(E_1^T A^T SAE_1 + \left(E_2^T \left(A_d + BYX^{-1}\right)^T SAE_1\right) \\ &+ E_1 AS\left(A_d + BYX^{-1}\right) E_2 \\ &+ E_2^T \left(A_d + BYX^{-1}\right)^T S\left(A_d + BYX^{-1}\right) E_2 \\ &- E_1^T SE_1 + \alpha_1^T N \alpha_1 - \alpha_2^T N \alpha_2 \\ &+ (d+1)\alpha_1^T M \alpha_1 - \alpha_3^T M \alpha_3 - \alpha_4^T M \alpha_4 \Big) \alpha_1 \delta(k) \\ &< -\delta(k)^T \alpha_1^T \left(\begin{bmatrix} E_3^T X & E_2^T Y^T \end{bmatrix} \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \right. \\ &\left. \times \begin{bmatrix} XE_3^T \\ YE_2 \end{bmatrix} \right) \alpha_1 \delta(k). \end{aligned} \tag{24}$$

The above inequality can be rewritten as

$$\begin{aligned} &\alpha_1^T \Big(E_1^T A^T SAE_1 + \left(E_2^T \left(A_d + BYX^{-1}\right)^T SAE_1\right) \\ &+ E_1 AS\left(A_d + BYX^{-1}\right) E_2 \\ &+ E_2^T \left(A_d + BYX^{-1}\right)^T S\left(A_d + BYX^{-1}\right) E_2 \\ &- E_1^T SE_1 + \alpha_1^T N \alpha_1 - \alpha_2^T N \alpha_2 \\ &+ (d+1)\alpha_1^T M \alpha_1 - \alpha_3^T M \alpha_3 - \alpha_4^T M \alpha_4 \Big) \alpha_1 \\ &< -\alpha_1^T \begin{bmatrix} E_3^T X & E_2^T Y^T \end{bmatrix} \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} XE_3^T \\ YE_2 \end{bmatrix} \alpha_1. \end{aligned} \tag{25}$$

By applying the congruence transformation to (25), we see that this is equivalent to

$$\begin{aligned} &\big(\Lambda^T X^T SX\Lambda + X\Gamma^T \Lambda + Y^T \Theta^T \Lambda + \Lambda^T \Gamma X + \Lambda^T \Theta Y \\ &+ (\Gamma X + \Theta Y)^T S(\Gamma X + \Theta Y) - \Phi^T X^T SX\Phi \\ &+ \alpha_1^T X^T NX\alpha_1 - \alpha_2^T X^T NX\alpha_2 \\ &+ (d+1)\alpha_1^T X^T MX\alpha_1 - \alpha_3^T X^T MX\alpha_3 - \alpha_4^T X^T MX\alpha_4\big) \\ &< -\alpha_1^T \begin{bmatrix} E_3^T X & E_2^T Y^T \end{bmatrix} \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} XE_3^T \\ YE_2 \end{bmatrix} \alpha_1. \end{aligned} \tag{26}$$

Substituting $S = gX_{-1}$, $N = gG_{-1}$ and $M = gW_{-1}$, into (26) and using Schur complement, we get

$$\begin{bmatrix} \widetilde{\alpha} & \alpha_1^T X^T & \alpha_2^T X^T & \alpha_3^T X^T & \alpha_4^T X^T & \alpha_1^T E_3^T X^T Q^{\frac{1}{2}} & \alpha_1^T E_2^T R^{\frac{1}{2}} Y^T R^{\frac{1}{2}} \\ * & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -N^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & N^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & M^{-1} & M^{-1} & 0 & 0 \\ * & * & * & * & * & gI & 0 \\ * & * & * & * & * & * & gI \end{bmatrix} < 0, \tag{27}$$

where $\widetilde{\alpha} \triangleq (d+1)\,\alpha_1^T \left(-2\mu X - \mu^2 M^{-1}\right)$.

By applying the Schur complement on $-2\mu X - \mu^2 M^{-1}$, we obtain (27), which ends the proof. □

4. SIMULATION RESULTS

In order to show the considerable contribution in the performance of the proposed control scheme, we consider a quarter vehicle active suspension systems [7, 18]. The quarter-vehicle suspension model parameters in Table 1 are used in this study.

$$\frac{d}{dt}\begin{bmatrix} z_{us}(t) - z_0(t) \\ \dot{z}_{us}(t) \\ z_s(t) - z_{us}(t) \\ \dot{z}_s(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_{us}}{m_{us}} & \frac{(c_s + c_{us})}{m_{us}} & \frac{k_s}{m_{us}} & \frac{c_s}{m_{us}} \\ 0 & -1 & 0 & 1 \\ 0 & \frac{c_s}{m_s} & -\frac{k_s}{m_s} & -\frac{c_s}{m_s} \end{bmatrix} \times \begin{bmatrix} z_{us}(t) - z_0(t) \\ \dot{z}_{us}(t) \\ z_s(t) - z_{us}(t) \\ \dot{z}_s(t) \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ \frac{1}{m_{us}} & \frac{c_{us}}{m_{us}} \\ 0 & 0 \\ -\frac{1}{m_s} & 0 \end{bmatrix} \begin{bmatrix} u(t) \\ \dot{z}_0(t) \end{bmatrix} \tag{28}$$

Table 1. Quarter-vehicle suspension model parameters.

Parameters	Description	Numerical value
m_s	Quarter body mass	972 [kg]
m_{us}	Unsprung mass	113.6 [kg]
k_s	Coefficient of suspension spring	42719.6 [N/m]
k_{us}	Coefficient of tire sprung	101115 [N/m]
c_s	Damping ratio of damper	1095 [N*s/m]
c_{us}	Damping of the pneumatic tire	14.6 [N*s/m]
N_s	The stiffness of the passive suspension	4271.96 [10% of k_s]

which can be further expressed by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bw(t) \\ y(t) = Cx(t) \end{cases} \tag{29}$$

where the state variable are the tire deflection $x_1(t) = z_{us}(t) - z_0(t)$, the unsprung-mass velocity $x_2(t) = \dot{z}_{us}(t)$, the suspension stroke $x_3(t) = z_s(t) - z_{us}(t)$, and the sprung-mass velocity $x_4(t) = \dot{z}_s(t)$. The coefficients of the state equation also can be presented in the following form:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_{us}}{m_{us}} & -\frac{(c_s+c_{us})}{m_{us}} & \frac{k_s}{m_{us}} & \frac{c_s}{m_{us}} \\ 0 & -1 & 0 & 1 \\ 0 & \frac{c_s}{m_s} & -\frac{k_s}{m_s} & -\frac{c_s}{m_s} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -1 \\ \frac{1}{m_{us}} & \frac{c_{us}}{m_{us}} \\ 0 & 0 \\ -\frac{1}{m_s} & 0 \end{bmatrix}$$

Note that the suspension system in (29) is a model with uncertainty in the sprung mass ms and the unsprung mass mu varies in the given ranges. In the meantime, the actuator delay should be taken into account, since the suspension performance could be affected by these factors. It leads to the system as

$$\begin{cases} \dot{x}(t) = A(k)x(t) + A_d(k)x(t-d_k) \\ \qquad + B(k)w(t-d_k) \\ y(t) = Cx(t) \end{cases} \tag{30}$$

In this simulation, we consider the discrete-time system with time-varying delay of an active car suspension (30), with sampling time $T = 0.1$ second. Then the system (1) is considered with the following data:

$$A = \begin{bmatrix} 0.9574 & 0.0093 & 0.0179 & 0.0005 \\ -8.3041 & 0.8489 & 3.4855 & 0.1073 \\ 0.0425 & -0.0093 & 0.9799 & 0.0094 \\ -0.0541 & 0.0125 & -0.4138 & 0.9873 \end{bmatrix}$$

$$A_d = \begin{bmatrix} 0.6985 & 0..0053 & 0.1217 & 0.0036 \\ -46.8014 & 0.1036 & 18.2011 & 0.5882 \\ 0.2880 & -0.0048 & 0 & 0.0060 \\ -3.7205 & 0.0687 & -2.6222 & 0.9172 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -0.0098 \\ 0.0001 & 0.0438 \\ 0 & -0.0001 \\ 0 & 0.0002 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}.$$

The controller parameters used in simulation are as follows: The weighting matrices are $Q = I$ and $R = 0.5I$, the delay bounds are $d_m = 1$, $d_M = 10$. For the power limit of the hydraulic actuator, the hard constraint on the active suspension is imposed $|u(k)| \le u_{\max}$ with $u_{max} = 1500$. In this study the sprung and the unsprung masses are assumed belonging to the ranges $[873, 1073]$ and $[104, 124]$, respectively.

By solving the convex optimization problem formulated in Theorem 1, the minimum guaranteed closed-loop performance obtained is $_{min} = 11,9729$.

To evaluate the suspension characteristics with respect to ride comfort, vehicle handling, and working space of the suspension, comparison results between the RMPC and the open-loop and MPC are proposed.

From these figures, the RMPC can still stabilize the

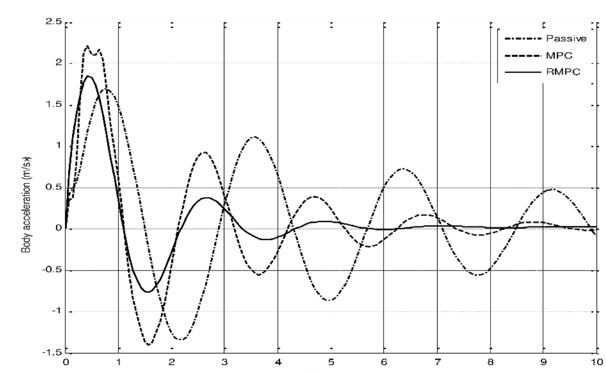

Fig. 1. Step response of Body acceleration.

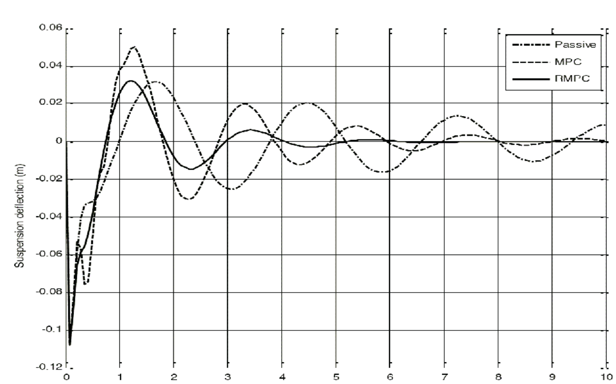

Fig. 2. Step response of suspension deflection.

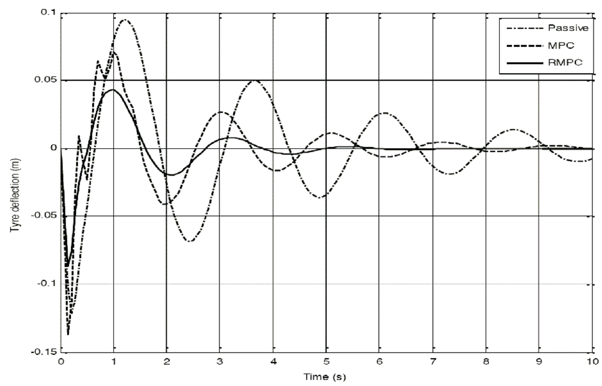

Fig. 3. Step response of tire deflection.

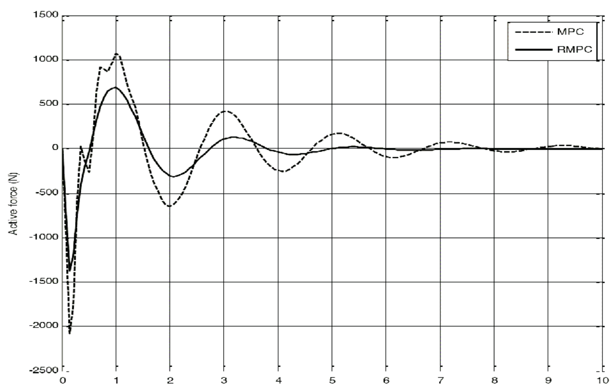

Fig. 4. Step response of active force.

closed-loop system with no degradation on performance. We can see that the RMPC controller yields the less value of the maximum body acceleration, compared with the passive suspension system and MPC controller.

Fig. 1 shows that the proposed algorithm can efficiently compensate for the time-varying delays and uncertainties, and has a good performance. Indeed, the fluctuation value of body acceleration is reduced by 59.78% at $t = 2.7$s compared with MPC and open-loop (66.36%), while the fluctuation value of suspension deflection is about 70% less than that of the MPC and 67.57% of the open-loop one at $t = 3.4$s. Moreover, the fluctuation value of tire deflection is about 68.50% less than that of the MPC controller and 84.00% of the open-loop one at $t = 3$s.

In addition, we can see from Fig. 4 that the active control force constraint is respected with the RMPC controller, which is not the case by MPC controller. Moreover the steady-state error approaches zero as well when $t = 5$s with RMPC.

5. CONCLUSION

In this paper, a robust model predictive control algorithm for polytopic uncertain systems with time-varying delays is presented. The goal is to design, at each sampling time k, a state feedback control law that minimizes an upper bound of the worst-case objective function, subject to constraints of control inputs. Sufficient conditions for the existence of the RMPC controller are proposed. Indeed, state feedback control law is obtained by solving an optimization problem using LMI terms. Simulation results have been given to illustrate the effectiveness of the proposed method.

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Sofiane Bououden received the B.Eng. and M.Eng. degrees in control systems from the Constantine University, in 2004 and 2006, respectively, and the Ph.D. degree in control systems from the Constantine University, in 2010. He is currently an assistant Professor with the department of Industrial Engineering, University of Khenchela, Algeria. His main research interests include fuzzy model, intelligent and predictive control, and robust adaptive control. He is an active reviewer for many international journals.

Mohammed Chadli received the Master degree (DEA) from the Engineering School INSA-Lyon in 1999, the PhD thesis from the CRAN-Nancy France in 2002 and his habilitation in 2011 at the University of Picardie Jules Verne (UPJV) in Amiens, France. From 1999 to 2004, he was an Assistant Prof CRAN-INPL (Institut National Polytechnique de Lorraine). Since 2004, he has been Associate Professor at the UPJV-MIS France. He was a visiting professorship at the Technical University of Ostrava-Czech Rep., University of Agder-Norway and the University of Shanghai-China. His research interests include fuzzy/LPV and switched systems, singular systems, robust control, fault detection and isolation (FDI), fault tolerant control (FTC) via LMI, SOS and Lyapunov methods. On the application side he is mainly interested in automotive control and renewable energy. He is author/co-author of 4 books, book chapters and numerous articles published in international journals and conferences. Dr. Chadli is a senior member of IEEE, he is serving as an Editorial Board member for various peer-reviewed journals including IET-CTA, Asian Journal of Control, Journal of Franklin Institute, and was a Guest Editor for Special Issues in international journals.

Lixian Zhang received the Ph.D. degree in control science and engineering from Harbin Institute of Technology, Harbin, China, in 2006. From January 2007 to September 2008, he was a Postdoctoral Fellow in the Department Mechanical Engineering at the Ecole Polytechnique de Montreal, Canada. He was a Visiting Professor at the Process Systems Engineering Laboratory, Massachusetts Institute of Technology, Cambridge, MA, USA, from February 2012 to March 2013. Since January 2009, he has been with Harbin Institute of Technology, where he is currently a Professor in the Research Institute of Intelligent Control and Systems. His research interests include nondeterministic and stochastic switched systems, networked control systems, and their applications. Dr. Zhang serves as an Associate Editor for various peer-reviewed journals, including the IEEE Transactions on Automatic Control, IEEE Transactions on Cybernetics, etc., and was a leading Guest Editor for the Special Section on Advances in Theories and Industrial Applications of Networked Control Systems in the IEEE Transactions on Industrial Informatics.

Ting Yang received the B.S. degree in automation from Tianjin University of Science and Technology, Tianjin, China, in 2009 and the M.S. degree in control science and engineering from the Harbin Institute of Technology, Harbin, China, in 2011, where she is currently working toward the Ph.D. degree in the Research Institute of Intelligent Control and Systems. From January 2013 to January 2015, she was a visiting scholar at Department of Mechanical and Aerospace Engineering, North Carolina State University, Raleigh, NC, USA. Her research interests include nondeterministic and stochastic switched systems, fuzzy control systems, model predictive control and their applications.