Application of Linear Programming in Production Management: A case study in SOFAMEC Khenchela

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Abstract:
In production management we face the problem of choice between several options: produce in the company in normal time, resorting to additional time, or subcontracting some productive operations andLinear programming represents an important tool in modeling a such problem. In this case study we essay to modelize this problem in order to elaborate the optimal production plan.The main conclusion of the paper is that the dispatching of the quantity to be produced on several times (normal and additional) and several actors (using the concerned enterprise facilities and the facilities of other enterprises through subcontracting) may reduce the total production cost significantly.
Keywords: Linear programming, production management, normal time production, additional time production, subcontracting.
Jel Classification Codes: **C61**

1. INTRODUCTION

Linear programming continues to develop, to address new areas of application to find solutions to problems encountered...and consequently to occupy a place increasingly important in the allocation of scarce resources and the rationalization of decision-making.

This technique has reached a maturity confirmed on the one hand by the existence of algorithms able to easily solve problems of considerable size and on the other hand, by a rich variety of applications (Teghem, 1996, p. 15). It occupies today an important place in the practice of different companies; Already in November 1978, a survey of 184 major US

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companies surveyed reveals that 133 or more than 72% use linear programming in production management and financial management decisions (Haji, 2003, p. 122).

Linear programming is a quantitative technique that consists in optimizing (maximizing or minimizing) the value of an objective expressed in a linear function called the 'objective function' subject to a set of conditions called 'constraints' which take the form of equations or linear inequalities.

In our case study that took place during the month of (February

2021) in the company sofamec; we will try to propose a methodology to improve the performance of the production function by the treatment of process selection problem.

1.1. Research Question

In this paper we shall treat the following problematic:

should diversification of production proccess reduce production total cost by application of linear programming?

1.2. Research hypotheses

Concerning the study **hypotheses** the main is: the application of linear programming in the problem of process selection should reduce the cost of production.

1.3 Objectives of the Research

The aim of the paper is to provide a practical guide on how to modelize a process selection problem.

The method of the study is the descriptif appraoch with quantitative case study by linear programming tool.

2.Literature review

When searching through the literature of applying linear programming in enhancing the function of production performance we remark at one hand the importance and on the other the variety of those applications.

An application of linear programming has been done by (Jyothi, 2019) in their paper entitled “Application of Linear Programming Model for Production Planning in an Engineering Industry-A Case Study”.

This paper develops a mathematical model for determining the best possible required capacity, workforce and lot-size. going through the existing practices of production planning in a single piece flow based cellular manufacturing unit producing auto electrical parts. It is a linear programming model with three objectives namely, Production cost minimization, production quantity maximization and maximization of capacity utilization. It is to be solved by considering each objective sequentially as a Lexicographic approach. The results obtained from the model are compared with actual observed values for validation.

in the entitled paper "Applying linear programming model to aggregate production planning of coated peanut products" (Rohmah, 2020) aimed to set the overall production level for each grade of coated peanut product to meet market demands with a minimum production cost. The linear programming model was applied in this study. The proposed model was used to minimize the total production cost based on the limited demand of coated peanuts. The demand values applied to the method was previously forecasted using time series method and production capacity aimed to plan the aggregate production for the next 6 month period. The results indicated that the production planning using the proposed model has resulted a better fitted pattern to the customer demands compared to that of the company policy. The production capacity of product family A, B, and C was relatively stable for the first 3 months of the
planning periods, then began to fluctuate over the next 3 months. While, the production capacity of product family D and E was fluctuated over the 6-month planning periods, with the values in the range of 10,864 - 32,580 kg and 255 – 5,069 kg, respectively. The total production cost for all products was 27.06% lower than the production cost calculated using the company's policy-based method.

(Masiyazi, 2019) had held a case study entitled "Productivity Improvement through Process Optimization: Case Study of a Plastic Manufacturing and Sales Company". The paper outlines the improvement of productivity through process optimization at a plastic manufacturing company. The research was a direct application of the concepts of plant layout, reliability centered maintenance and Computer Integrated Manufacturing (CIM) in which focus was given to production facilities and manufacturing support systems. Modeling and simulation, Pareto analysis, root cause analysis, Weibull analysis, time study, experimentation, interviews, and historical data were used as research and analysis instruments. The production facilities were optimized through the application of optimization tools. The manufacturing support system was optimized through the design of a computerized manufacturing support system that automated the various business functions which include batching, production planning, inventory management and maintenance planning and scheduling. The research revealed that there is an important link between the various manufacturing systems (organization of people and facilities) within a company and that these need to be integrated by a computerized manufacturing support system for efficient and effective operation. It was recommended that companies need to adopt CIM systems since they open a good platform for higher productivity within an organization through automation of manufacturing systems and computerization of business functions to reduce manual labor. However, whenever a company wishes to adopt such a system, it is a good idea if the system is specially developed and customized for that particular company only as this will make it easy to implement and monitor.

(Uzorh, 2013) had done a case study using integer linear programming in production planning. In his research; a mathematical model of textile production industry's problem was developed concerning the united Nigerian textiles which is one of the latest and biggest textile industries in Nigeria. Annually textile utilization in Nigeria increases therefore its production should increase also. Generally, the net profit may increase by increasing sell price or reducing total cost of textile production. The problem studied was how much of the raw cottons and polyester will be transformed into the valuable textile per day and the final aim is to increase profit.

In the case study entitled "A case study application of linear programming and simulation to mine planning" held by (Carvalho, 2012) had been analyzed the impact of the uncertainty associated with the input parameters in a mine planning optimization model. A real example was considered to aid in the building of a mathematical model that represents the coal production process with reference to the mining, processing, and marketing of coal. This model was optimized using the linear programming concept whereby the best solution was perturbed by the stochastic behavior of one of the main parameters involved in the production process. The analysis of the results obtained permitted an evaluation of the risk associated with the best solution due to the uncertainty in the input parameters.

In their paper entitled “A simulation case study of production planning and control in printed wiring board manufacturing”; (Korhonen, 2019) had held a simulation project concerning production planning and control in printed wiring board (PWB) manufacturing. This issue becoming more difficult as PWB’s technology is developing and the production routings become more complex. Simultaneously, the strategic importance of delivery accuracy, short delivery times, and production flexibility is increasing with the highly fluctuating demand and short product life cycles of end products. New principles, that minimize throughput time while guaranteeing excellent customer service and adequate capacity utilization, are needed for production planning and control.

Simulation is needed in order to develop the new principles and test their superiority. This paper presents an ongoing simulation project that aims at developing the production planning and control of a PWB manufacturer. In the project, a discrete event simulation model is built of a pilot case factory. The model is used for comparing the effect of scheduling, queuing rules, buffer policies, and lot sizes on customer service and cost efficiency.

In our study the aim doesn’t concern the increasing of profit by selection between raw materials involved in production; but is to study the possibilities of production in additional time and by subcontracting in order to reduce the total cost of production.

3. Presentation of the applied tool: linear programming

In the context of the application of linear programming; The modeling of the problem will give what is called 'linear program'; which consists in general of: $X_j \in \{0,1\}$

3.1 Objective function

(it is also called 'economic function') which expresses algebraically the objective one wants to achieve; This is the function for which we are looking for the optimal value (maximum or minimum).This economic function will be evaluated, as the case may be, in francs, time, energy, distance, etc(Kaufmann, 1970, p. 11).

3.2 The constraints

these are the limitations imposed by the scarce resources and the different conditions that must be respected by the solution that will be proposed.

3.3Non-negativity constraints

these are constraints that indicate that the decision variables must be null or positive; given the impossibility of assigning negative values to the quantities to be produced, for example.

The concept of linearity represents the major hypothesis of this technique; This means: the productions and the consumptions of the activities are additive; this assumption is tantamount to neglecting the economies and the losses that may result from the simultaneous use of more than one activity (Essid, 2001, p. 24).

They are therefore the two traditional hypotheses of linear analysis: multiplication by a scalar and additivity.

Concerning the resolution of linear programs; the so-called 'Simplex' method is mentioned, the general principle of which is to move through several iterations 'so as to improve the economic function each time (Lacaze, 1990, p. 25); however as our approach tends more towards the practical aspect than towards the explanation of the resolution of a linear program; we will not examine the details of this algorithm.

the utility of linear programming is expanding to address a growing variety of problems; through the extensions she has known; and the nature of the decision variables it may contain, and in particular: linear integer programming and that in binary variables.

In reality ; all or certain decision variables in the program can only take integer values (Haji, 2003, p. 146) ; for example, can we assign the value (2.5) to a variable that indicates the number of chairs to be produced? In addition; Binary variables can be seen as integer variables subject to the constraint of belonging to the interval [0,1] (Nobert, 2001, p. 289) ; and so the constraint:

$$X_j \in \{0,1\}$$

Can be written:

$$X_j \leq 1$$
$$X_j \geq 0$$
$$X_j : \text{Integer}$$

The decisions relating to the binary variables are varied: whether or not to execute a command or a task, to dedicate or not a resource to the satisfaction of a request or not to leave a location ‘A’ to go to a location’B ‘...(Giard, 1998, p. 10).
The introduction of integer linear models and those in binary variables made it possible to deal with more complicated problems than the classical problems treated by linear programming.

The application of this technique addresses several areas of which the main ones are:

a. Mixing problems

these are the problems in which one seeks to mix or extract ingredients from raw materials so as to respect quality standards (ingredient contents) and minimize the total cost of production (Guéret, 2000, p. 81) ; the best-known examples are: the production of foods that respect certain nutritional conditions, the manufacture of alloys in metallurgy and the refining of petroleum products.

b-Production management

An example of such importance can be cited here in practice; this is the problem of choosing 'production processes': which arises when a company can use overtime in all or some of these workshops; which leads to an increase in cost, or when it can outsource all or some operations of production; which will reveal - for a given product - two parameters (costs or profits ..); one relating to the internal product and the other relating to the same product if it is produced by another person in the context of "subcontracting"; which amounts to distinguishing for the same product four production lines corresponding to four costs (cost of the internal production at the normal time, overtime, cost of the external production at the normal time, overtime).

This is a type of problem known in the Anglo-Saxon literature of operational research under the name of 'process selection problem'; and is normally characterized by:

- A level of production imposed for different products;
- Several possible production processes for at least one product;

the production processes of a product differ in the technical processes used or the resources consumed (machines of different technical or economic performance, call for overtime, other qualifications of staff, subcontracting, etc.) but in all case, the final physical product is the same;

- The unit costs and factors used depend on the chosen process;
- The problem is that of determining, for each product, the quantity produced by each process, which minimizes the cost of production; if the selling price is constant, this criterion is equivalent to that of maximizing the margin on variable cost;

- This definition of the optimal program must take into account the available endowments of the various productive factors used in the selected processes (Giard, 1998, pp. 19-20).

c- Assignment problems

These are problems that relate to a special category of linear programs in which the economic function is to assign a number of sources (or origins) to the same number of destinations at a minimum cost (Benghezal, 2000, p. 159).

4 - Presentation of the studied company

"SOFAMEC" is a company with a collective name created in 1992; a company of mechanical manufacturing and metal furniture located at the industrial zone of Khenchela;

implanted on a ground of an area of 3000m2 including 1360 m2 covered in workshops and administration.

its main purpose is to meet the needs of local authorities and local administrations in the field of urban furniture; school equipment, playground equipment; metal fences; laboratory benches...(SOFAMEC, 2020)

it produces a large set of metal products among them we site some examples (see appendix).

5 – Problem statement

by means of our interview with the technical director we remarqued that during some previous periods the question of how to meet customers' orders in the ideal time is frequently asked, on the other side we concluded that some productive processes can be subcontracted with external dealers (welders especially) and consequently that might reduce cost of production. In order to test this hypothesis we chose two orders; one concerning "laboratory benches" (see: figure 5) the other concerns "bus shelters" (see: figure 6).

6 – Data collection

Via our interview with the technical director we obtained the information about those two products as detailed in the following table:

Table 1. cost elements of the studied products

output / Input	Laboratory bench		Bus shelter		Availablequantity/ month
	Qty	cost	Qty	cost	
square tube 100/100 (linearmeter)	0	0	4.4	3000	700
square tube40/40(lm)	0	0	10	2700	80
Rectangular tube 100/50(lm)	0	0	3	1500	600
Rectangular tube 40/30(lm)	25	3800	28	4200	3000
Resinsheet (m^2)	1.2	1200	7.2	7200	218
Multiple wood (m^2)	1.2	600	0	0	200
MDF Wood (m^2)	1.8	1000	0	0	200
polycarbonate (m^2)	0	0	5.4	7600	700
Rectangular tube 60/30(lm)	0	0	6	1800	700
stainless tub	1	2200	0	0	100
Laboratorytap (unit)	1	600	0	0	100
Gas stop valve(unit)	1	900	0	0	100
electricity socket (unit)	2	300	0	0	200
Perclose (lm)	2	400	0	0	600
electricwire 2*2.5 (lm)	5	250	0	0	100
Fixing sole (unit)	0	0	4	1800	800

Necessary production time (hour)	72	72	
Time cost (dinars/hour)	188	188	
Total time cost	13536	13536	
Other costs (energy…) in Dinars	4000	2500	
Production cost per product	28786.00	45836.00	

Source. elaborated by the author according to his interview with the technical director

In the table above we remark that the production cost of a laboratory bench is (28786.00 dinars) while a “bus shelter” costs (45836.00 dinars); but there will be more possibilities to execute the work:

- The company produces by using its own resources in normal time;
- The company produces by using its own resources and resorting to supplementary hours;
- The company subcontracts some operations (welding especially) to external leaders;

In fact for each possibility there will be a particular production cost; so we have to define for each product three costs (internal production in normal time, internal production in additional time, the cost of the product when subcontracting some operations).

In SOFAMEC the week-end is (02) days per week which can be exploited as additional time; thus there will be:
(02*4 = 8) eight days per month; multiplied by the number of workers (10) and the number of daily work hours (08) gives: (640) hours per month.
Concerning the cost of the additional time; it is calculated by multiplying the cost of normal time by the coefficient (1.5), therefore the cost of a laboratory bench produced in the additional time will be:
(28786.00 - 13536.00) + (13536.00*1.5) = 35554.00
Concerning the bus shelter the cost will be:
(45836.00 - 13536.00) + (13536.00*1.5) = 52604.00

Concerning the cost of production by an external welder the necessary resources remain the same; the only change is simply in some elements such as fix cost and cost of labor, but in general the referential welder estimated that the cost will be (+10%) compared with the cost of internal production in normal time; the following table resumes the different costs:

Table 2. cost of production in each case

Process product	Internal prod-uction (normal time	Internal prod-uction additional time	Subcontracted production
Labo bench	28786.00	35554.00	31664.60
Bus shelter	45836.00	52604.00	50419.60

Source. elaborated by the researcher

Now the question is how many unit to produce in each process?

This means to determine the values of six (06) variables:

Table 3. matrix of variables

Process product	Internal prod-uction (normal time	Internal prod-uction additional time	Subcontracted production
Labo bench	X_{11}	X_{12}	X_{13}
Bus shelter	X_{21}	X_{22}	X_{23}

Source. elaborated by the researcher

For example:

X_{11}: number of labo benches produced by the company in normal time;

X_{12}: number of labo benches produced by the company in additional time;

X_{13}: number of labo benches produced by subcontracting.

7 – Modeling

Our objective is to minimize the total cost of production; thus we have the objective function as follows:

$$Min:(Z) = 28786x_{11} + 35554x_{12} +$$
$$31664x_{13} + 45836x_{21} + 52604x_{22} + 50419x_{23}$$

This function is subject to a set of constraints:

- The available quantities of resources: referring to (table.1) we deduce the following constraints:

$4.4x_{21} + 4.4x_{22} + 4.4x_{23} \leq 700$

$10x_{21} + 10x_{22} + 10x_{23} \leq 80$

$3x_{21} + 3x_{22} + 3x_{23} \leq 600$

$25x_{11} + 25x_{12} + 25x_{13} + 28x_{21} + 28x_{22} + 28x_{23} \leq 3000$

$1.2x_{11} + 1.2x_{12} + 1.2x_{13} + 7.2x_{21} + 7.2x_{22} + 7.2x_{23} \leq 218$

$1.2x_{11} + 1.2x_{12} + 1.2x_{13} \leq 200$

$1.8x_{11} + 1.8x_{12} + 1.8x_{13} \leq 200$

$5.4x_{21} + 5.4x_{22} + 5.4x_{23} \leq 700$

$6x_{21} + 6x_{22} + 6x_{23} \leq 700$

$x_{11} + x_{12} + x_{13} \leq 100$

$x_{11} + x_{12} + x_{13} \leq 100$

$x_{11} + x_{12} + x_{13} \leq 100$

$2x_{11} + 2x_{12} + 2x_{13} \leq 200$

$2x_{11} + 2x_{12} + 2x_{13} \leq 600$

$5x_{11} + 5x_{12} + 5x_{13} \leq 100$

$4x_{21} + 4x_{22} + 4x_{23} \leq 800$

Concerning the time constraint of normal time production:

$72x_{11} + 72x_{21} \leq 1200$

And for additional time:

$72x_{12} + 72x_{22} \leq 640$

In addition we have another set of constraints concerning the demand (an order of 18 benches, and another of 08 bus shelters):

$x_{11} + x_{12} + x_{13} \geq 18$

$x_{21} + x_{22} + x_{23} \geq 8$

And finally we impose on our variables to be integer:

$x_{ij} \in \square$

8 – Model solving and discussion of results

Using the software "STORM" we solve the model and obtain the following results:

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OPTIMAL SOLUTION - SUMMARY REPORT

Variable	Value	Cost	Lower bound	Upper bound
X11	8	28786.0000	0	Infinity
X12	0	35554.0000	0	Infinity
X13	10	31664.0000	0	Infinity
X21	8	45836.0000	0	Infinity
X22	0	52604.0000	0	Infinity
X23	0	50419.0000	0	Infinity

Objective Function Value = 913616

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OPTIMAL SOLUTION - SUMMARY REPORT

Constraint	Type	RHS	Slack
CONSTR 1	<=	700.0000	664.8000
CONSTR 2	<=	80.0000	0.0000
CONSTR 3	<=	600.0000	576.0000
CONSTR 4	<=	3000.0000	2326.0000
CONSTR 5	<=	218.0000	138.8000
CONSTR 6	<=	200.0000	178.4000
CONSTR 7	<=	200.0000	167.6000
CONSTR 8	<=	700.0000	656.8000
CONSTR 9	<=	700.0000	652.0000
CONSTR 10	<=	100.0000	82.0000
CONSTR 11	<=	100.0000	82.0000
CONSTR 12	<=	100.0000	82.0000
CONSTR 13	<=	200.0000	164.0000
CONSTR 14	<=	600.0000	564.0000
CONSTR 15	<=	100.0000	10.0000
CONSTR 16	<=	800.0000	768.0000
CONSTR 17	<=	1200.0000	48.0000
CONSTR 18	<=	640.0000	640.0000
CONSTR 19	>=	18.0000	0.0000
CONSTR 20	>=	8.0000	0.0000

Objective Function Value = 913616

As given above the model indicates the following results:

Table 4. results of the model

Process product	Internal production (normal time	Internal production additional time	Subcontracted production
Labo bench	$X_{11}=08$	$X_{12}=0$	$X_{13}=10$
Bus shelter	$X_{21}=08$	$X_{22}=0$	$X_{23}=0$

Source. elaborated by the researcher

In the table above we remark that the optimal solution indicates to dispatch the ordered quantity of labo benches (18 unities) to be produced by the firm facilities in normal time (08) and subcontract the rest (10) to be produced out of the firm; thus it is irrational to make the firm's facilities operating for additional time ($X_{12}=0$).

Concerning the bus shelters; the model indicates to produce the total quantity (08 units) in the normal time inside the firm; and thus not resorting neither to additional time in the firm nor to external dealer by the means of subcontracting ($X_{22}=0$, $X_{23}=0$)

9. Conclusion

9-1 results of the study: Linear programming is an important tool in resource allocation and decision making; as it serves to enhance the performance in the companies and simplifies the deal with difficult problems.

In production management; linear programming can help in process selection efficiently; by simplifying the problem of choice between the normal time; additional time and subcontracting , by consequence elaborating the ideal production program.

9-2 recommendations:

The recommendation that we address to the company responsible is to take into consideration the possibility of executing work in different processes not only in internal normal time production; but it is possible to produce some quantities also in additional time or by subcontracting through the following steps:

- Determine the available additional time for production (640 hours in our case);
- Calculate the additional cost when producing in the additional time; by consequence the unitary cost of production in additional time (28786.00+676800 = 35554.00);
- Determine the available subcontractors of some productive processes and the relevant cost both in normal and additional time;
- Consequently there will be for a unit to be produced four production costs: cost of internal production in normal time, cost of internal production in additional time, cost of external production in normal time and cost of external production in additional time as shown before :

Process product	Internal production (normal time	Internal production additional time	Subcontracted production
Labo bench	28786.00	35554.00	31664.60
Bus shelter	45836.00	52604.00	50419.60

- Then there will be four variables to determine their values under the constraints of each case.

Appendix

Fig 1. Art fencing

Source: technical department, Sofamec

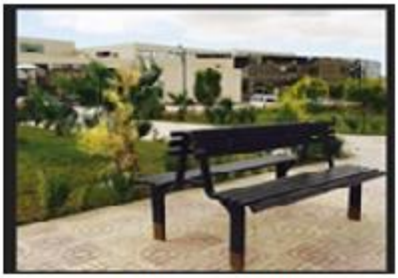
Fig 2. Spiral staircase

Source: same

Fig 3. Art ironwork

Source: same

Fig 4. Street furniture



Source: same

Fig 5. Laboratory bench

Source: same

Fig 6. Bus shelter

Source: same

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