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Experimental Study and Modelling of the Ferritic Steel Anisotropic Behavior under Associated and Non-Associated Flow Rule

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Abstract. In the present study, two different yield criteria were investigated to model and compare the yield thresholds functions for the plastic behavior of rolled sheets. These two different yield criteria as described via *Hill48* yield quadratic and F. Barlat *Yld2000-2d* non-quadratic criterion. For this purpose, an experimental device of simple tensile test and the studied material are described. The experimental results in terms of Yield stress and Anisotropic coefficient are estimated from the Associated Flow Rule (AFR) and Non-Associated Flow Rule (NAFR). However, it is found that the criterion of *Yld2000-2d* is the most appropriate model in comparison with the experimental results.

Introduction

During the forming process, the experimental study of the material's anisotropic behavior is complex, it requires preparation time and resources that are often very expensive. Thus, the industrialists are increasingly turning to numerical simulations by using numerical implantation and mathematical modeling by developing calculation models which predict results that are more or less consistent with the physics of transformation, and better characterize the behavior of the material.

The associated flow rule (*AFR*) is also referred to as the normality rule, by which the yield function is equal to the flow potential, while the non-associated flow rule (*NAFR*) is indicating that the yield function and the flow potential of the plasticity model are defined by two separate functions. Numerical studies suggested the use of the non-associated flow rule (*NAFR*) models to describe the anisotropic behavior of metals [1-3] due their efficiency and simplicity in addition to the fundamental theoretical and experimental arguments.

Recently, the attention towards the non-associated quadratic anisotropic models increased, even with the big issue of the experimental explanation of its applications [4].

For this purpose and in aim order to describe the anisotropic behavior of materials, several such functions (quadratic and non-quadratic) have been proposed. Initially, Hill [5], which is considered as a simple modification of the standard von Mises plasticity to give a quadratic yield function. As for the constitutive law, a more flexible and adaptable model is the non-quadratic anisotropic yield function, *Yld2000-2d* [6], was used to describe the initial anisotropic yield surface along with the isotropic hardening law for the yield surface evolution.

Experimental Procedure

The material used in this study was ferritic steel, known as *DIN 1623 St-14* (ASTM A620; NE 10130-2006). It was supplied by Algerian Tractors Company (ATC) as 1.35 mm annealed rolled sheet with a chemical composition listed in Table 1. Uniaxial tests were carried out to characterize its mechanical properties. Specimens were cut along 0°, 15°, 30°, 45°, 60°, 75° and 90° from the rolling direction (0°). while the equibiaxial tension (σ_b) was calculated. Thus, , $\sigma_b = \frac{\sigma_0+\sigma_{90}}{2}$.

Table 1 Mechanical properties of *DIN St-14* for 0°,45° and 90°

Direction	σ_m (MPa)	r
0°	255	1.6
45°	262	1.9
90°	258	1.8

Modelling of Anisotropic behavior under Yield Criteria

Hill48 yield criteria: The homogenous quadratic criterion proposed by Hill was the first and the most often yield function employed anisotropic models for sheet rolled. It's defined from equation. 1 by:

$$\sigma_0 = \sqrt{F(\sigma_{22}-\sigma_{33})^2 + G(\sigma_{33}-\sigma_{11})^2 + H(\sigma_{11}-\sigma_{22})^2 + 2L{\sigma_{23}}^2 + 2M{\sigma_{13}}^2 + 2N{\sigma_{12}}^2} \qquad (1)$$

F, G, H, L, M and N represent *Hill's48* anisotropic coefficients. In this study, *Hill's48* anisotropic coefficients were determined using two methods, the first one is based on yield stresses and *r-values* combined [7]:

$$F = \frac{2\sigma_0^2}{\sigma_{90}^2(1+r_{90})} \qquad G = 2 - \frac{2\sigma_0^2 r_{90}}{\sigma_{90}^2(1+r_{90})} \qquad H = \frac{2\sigma_0^2 r_{90}}{\sigma_{90}^2(1+r_{90})} \qquad N = \frac{4\sigma_0^2}{\sigma_{45}^2} - 1 + \frac{\sigma_0^2(r_{90}-1)}{\sigma_{90}^2(1+r_{90})} \qquad (2)$$

and the second one on yield stresses or *r-values:*

$$2F = \frac{\sigma_0^2}{\sigma_{90}^2} - 1 + \frac{\sigma_0^2}{\sigma_b^2} \qquad 2G = 1 - \frac{\sigma_0^2}{\sigma_{90}^2} + \frac{\sigma_0^2}{\sigma_b^2} \qquad 2H = 1 + \frac{\sigma_0^2}{\sigma_{90}^2} - \frac{\sigma_0^2}{\sigma_b^2} \qquad 2N = \frac{4\sigma_0^2}{\sigma_{45}^2} - \frac{\sigma_0^2}{\sigma_b^2} \qquad (3)$$

$$F = \frac{r_0}{r_{90}(1+r_0)} \qquad G = \frac{1}{(1+r_0)} \qquad H = \frac{r_0}{(1+r_0)} \qquad N = \frac{(1+2r_{45})(r_0+r_{90})}{2r_{90}(1+r_0)} \qquad (4)$$

Yld200-2d yield criteria: The yield criterion *Yld2000-2d* proposed by Barlat et al. one of the most accurate anisotropic yield functions for rolled sheets presents the following form:

$$2{\sigma_0}^k = |S_1' - S_2'|^k + |2S_2'' + S_1''|^k + |2S_1'' + S_2''|^k \qquad (5)$$

where S_i' and S_j'' denote the principal values of the tensors as:

$$\begin{bmatrix} S_{11}' \\ S_{22}' \\ S_{12}' \end{bmatrix} = \begin{bmatrix} C_{11}' & C_{12}' & 0 \\ C_{21}' & C_{22}' & 0 \\ 0 & 0 & C_{66}' \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{22} \\ s_{12} \end{bmatrix}, \quad \begin{bmatrix} S_{11}'' \\ S_{22}'' \\ S_{12}'' \end{bmatrix} = \begin{bmatrix} C_{11}'' & C_{12}'' & 0 \\ C_{21}'' & C_{22}'' & 0 \\ 0 & 0 & C_{66}'' \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{22} \\ s_{12} \end{bmatrix} \qquad (6)$$

s_{11}, s_{22} and s_{12} are the deviatoric stress components of the *Cauchy* stress. By solving the nonlinear equation from experimental data, the anisotropic coefficients (α_1–α_8) can be determined. The exponent *K* is taken to be six since the crystal of the ferritic steel sheets is *BCC* lattice.

Results and Discussion

For each direction, the yield stresses and *r-values* was normalized by the yield stress and *r-value* according to the rolling direction, the input data for the yield functions and the calculated anisotropic coefficients are listed in Tables 2-5. *Hill's48* four anisotropic coefficients (F, G, H and N) under Associated Flow Rule (*AFR*) were calculated from the Eq. 2, while using Eq. 4 and Eq. 5 were used under Non-Associated Flow Rule (*NAFR*). Although, the eight anisotropic coefficients of *Yld2000-2d*, were computed using MATLAB code. Newton-Raphson iteration method under

Associated Flow Rule (*AFR*), and Levenberg-Marquardt optimization method under Non-Associated Flow Rule (*NAFR*)

Table 2 Yield stress and *r*-values normalized by rolling direction input data

	r_0/r_0	r_{45}/r_0	r_{90}/r_0	r_b/r_0
r-values	1	1.461	1.125	0.718
	σ_0/σ_0	σ_{45}/σ_0	σ_{90}/σ_0	σ_{90}/σ_0
Yield Stress	1	1.027	1.011	1.005

Table 3 Anisotropic coefficients for *Hill48* and *Yld2000-2d* under Associated Flow Rule (*AFR*)

Hill48	**F**	**G**	**H**	**N**
	0.3382	0.3573	0.6426	1.7661

YLD2000-2d	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
K=6	1.0517	1.0413	1.1286	0.9684	0.9513	0.9967	1.0024	0.8380

Table 4 Anisotropic coefficients for *Hill48* and *Yld2000-2d* under (*NAFR_ r*-values)

Hill48	**F**	**G**	**H**	**N**
	0.3422	0.3496	0.6503	1.3838

YLD2000-2d	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
K=6	-0.3442	1.9523	1.2762	0.7881	0.7766	0.3409	0.7394	1.2252

Table 5 Anisotropic coefficients for *Hill48* and *Yld2000-2d* criterions under (*NAFR*_σ) based on yield stresses

Hill48	**F**	**G**	**H**	**N**
	0.4864	0.5055	0.4944	1.3630

YLD2000-2d	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
K=6	1.0659	1.0678	1.8808	0.5322	-0.578	-1.871	0.7635	1.6021

Fig. 1-3 illustrated the distribution of normalized yield stresses σ(θ), *r-values* r(θ), yield loci under Associated Flow Rule (*AFR*) and Non-Associated Flow Rule (*NAFR*) and also the combining yield stresses and *r-values.* As shown in Fig. 1 (a) and (b), when *Yld2000-2d* criterion, ensures a certain harmony and consistency concerning the evolution of experimental and theoretical curves. Fig. 2 (a) and (b) shows that the two criterions under Non-Associated Flow Rule are almost similar and in a good shape with the experimental one, although, the *Yld2000-2d* model describes perfectly the normalized yield stress and *r-values* anisotropy. Therefore, the measured yield loci of the material for the *Hill48* and *Yld2000-2d* criterions under Associated and Non-Associated Flow Rule was plotted in Fig. 3 (a) and (b). Both of the criterions show a remarkable anisotropic behavior, particularly under (*NAFR-r*) condition.

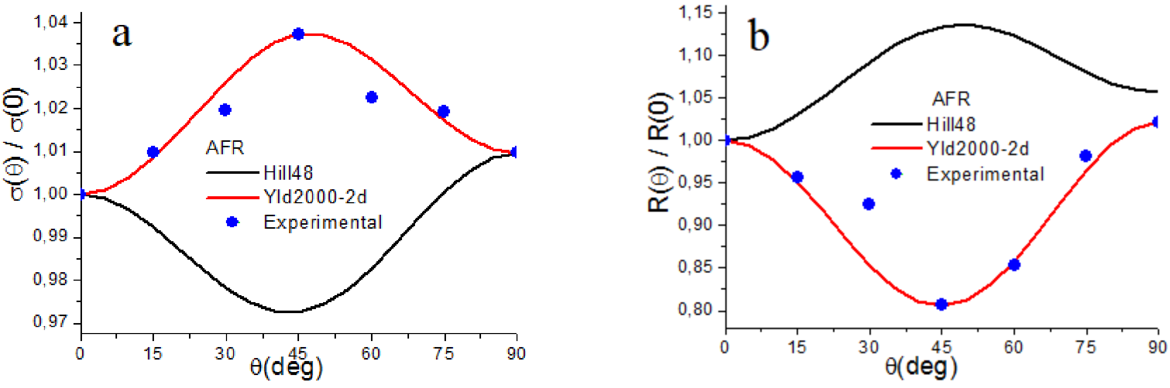

Figure 1 Comparison of normalized yield stress (a) and *r*-values (b) for *DIN St14* between experiments, *Hill48* and *Yld2000-2d* predictions under Associated Flow Rule (*AFR*)

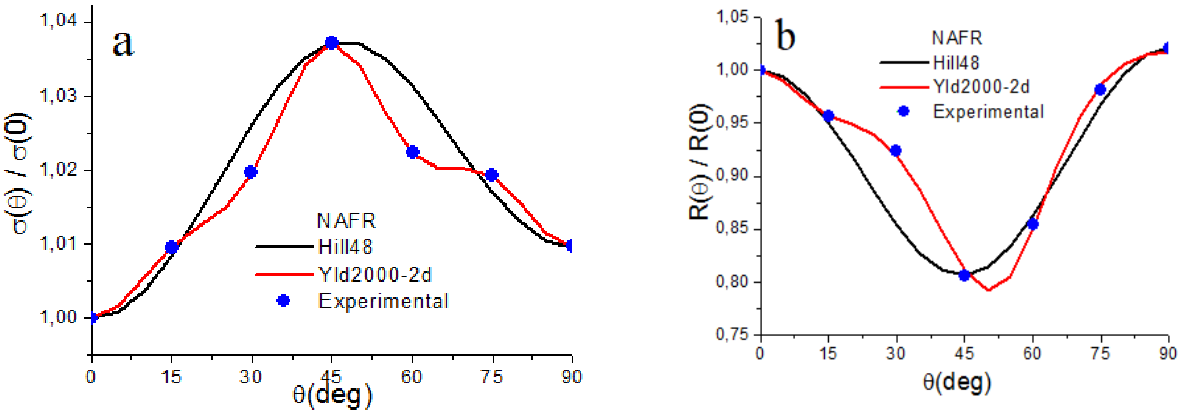

Figure 2 Comparison of normalized yield stress (a) and *r*-values (b) for *DIN St14* between experiments, *Hill48* and *Yld2000-2d* predictions under Non-Associated Flow Rule (*NAFR*)

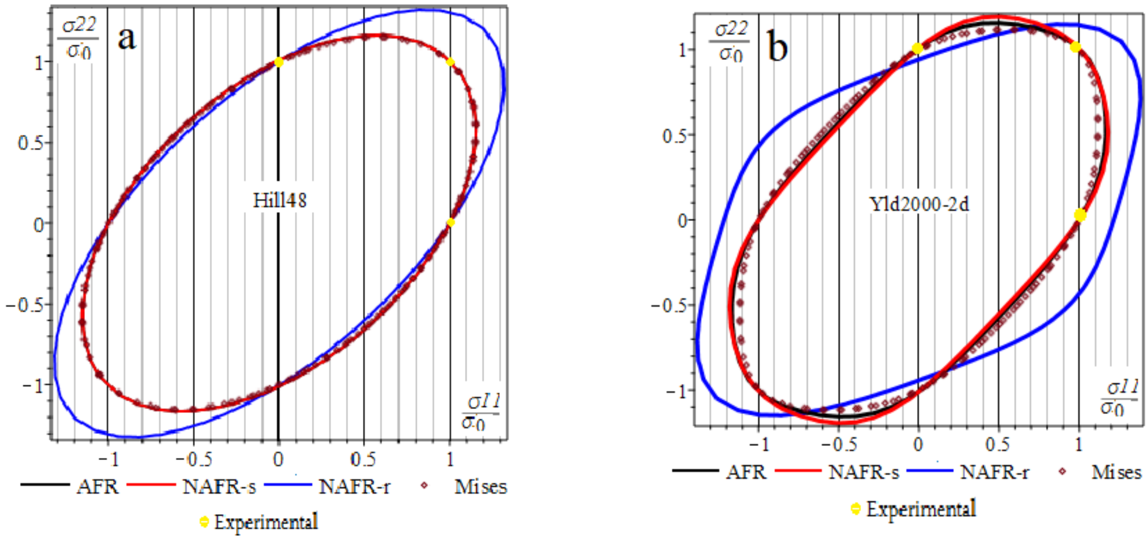

Figure 3 Comparison of yield loci for *DIN St14* between experiments, *Hill48* (a) and *Yld2000-2d* (b) under Associated Flow Rule (*AFR*) and Non-Associated Flow Rule (*NAFR*)

Summary

In this work, anisotropic behavior of ferrtic steel *DIN St-14* was investigated via two yield criterions, *Hill's48* and F. Barlat et al. (*Yld2000-2d*) criterions under Associated Flow Rule (AFR) and Non-Associated Flow Rule (NAFR). As conclusion, it was demonstrated that *Yld2000-2d* predicts the anisotropic mechanical behavior of *DIN St-14* steel better than *Hill's48* criterion in comparison with the both flow rules. In addition, The Non-Associated Flow Rule (*NAFR*) has indicated a better approach to the experimental results for both criterions.

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