

Exercise 1:

An optical fiber is made of a core with refractive index $n_1 = 1.48$ and a cladding with refractive index $n_2 = 1.46$. A light ray is launched from air ($n_0 = 1$) into the fiber.

1. Calculate the critical angle θ_c at the core–cladding interface.
2. Determine the numerical aperture (NA) of the fiber.
3. Calculate the maximum acceptance angle θ_{\max} in air for which light is guided inside the fiber.
4. A light ray enters with an angle $\theta = 10^\circ$. Determine whether it will be guided in the fiber.

Exercise 2:

A communication system transmits a signal using amplitude modulation (AM). The carrier has a frequency of 100 kHz and an amplitude of 5 V , while the message signal is a sinusoid of frequency 1 kHz and amplitude 2 V . The system first generates an AM signal. This signal is then passed through a nonlinear device followed by a frequency-sensitive converter, which transforms amplitude variations into frequency variations, producing an FM signal.

1. Write the mathematical expression $S(t)$ of the AM DSBFC signal using the given numerical values. Give the temporal and spectral representation according to the modulation index. Calculate the bandwidth of this signal.
2. Write the form of the resulting FM signal $S_1(t)$, where the amplitude is constant and the frequency varies with the message signal.
3. If the frequency sensitivity is $k_f = 2\text{ kHz/V}$, calculate:
 - the frequency deviation
 - the bandwidth of the FM signal

Exercise 3:

An analog signal $x(t)$ has a maximal frequency of $f_{\max} = 3\text{ kHz}$. It is sampled with a sampling period T_s and quantized using a 2-bit quantizer.

1. Calculate the sampling period T_s .
2. Explain briefly the sampling and quantization process.
3. Give the binary sequence corresponding to the digital signal $x[n]$.
4. Draw the Manchester coding of the obtained binary sequence.

Solution

Exercise 1

1.

$$\sin(\theta_c) = \frac{n_2}{n_1} = \frac{1.46}{1.48} = 0.9865$$

$$\theta_c = \arcsin(0.9865)$$

$$\theta_c \approx 80.9^\circ$$

2.

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$NA = \sqrt{1.48^2 - 1.46^2}$$

$$NA = \sqrt{2.1904 - 2.1316}$$

$$NA = \sqrt{0.0588}$$

$$NA \approx 0.24$$

3.

$$\sin(\theta_{\max}) = NA$$

$$\theta_{\max} = \arcsin(0.24)$$

$$\theta_{\max} \approx 13.9^\circ$$

4.

$$10^\circ < 13.9^\circ$$

So, The ray is guided

Exercise 2: Solution: AM \rightarrow FM Conversion

1. AM signal, spectrum and bandwidth

Given:

$$A_c = 5V, \quad A_m = 2V, \quad f_c = 100kHz, \quad f_m = 1kHz$$

Modulation index:

$$= \frac{A_m}{A_c} = \frac{2}{5} = 0.4$$

Message signal:

$$m(t) = 2 \cos(2\pi \cdot 1kHz \cdot t)$$

$$S(t) = 5 [1 + 0.4 \cos(2\pi \cdot 1 \text{ kHz} \cdot t)] \cos(2\pi \cdot 100 \text{ kHz} \cdot t)$$

Spectrum:

$$f_c = 100 \text{ kHz}, \quad f_c - f_m = 99 \text{ kHz}, \quad f_c + f_m = 101 \text{ kHz}$$

$$B_{AM} = 2f_m = 2 \times 1 \text{ kHz} = 2 \text{ kHz}$$

2. FM signal

$$S_1(t) = 5 \cos \left(2\pi f_c t + 2\pi k_f \int m(t) dt \right)$$

$$m(t) = 2 \cos(2\pi \cdot 1 \text{ kHz} \cdot t)$$

$$\int m(t) dt = \frac{2}{2\pi \cdot 1 \text{ kHz}} \sin(2\pi \cdot 1 \text{ kHz} \cdot t)$$

$$S_1(t) = 5 \cos(2\pi \cdot 100 \text{ kHz} \cdot t + \beta \sin(2\pi \cdot 1 \text{ kHz} \cdot t))$$

where:

$$\beta = \frac{k_f A_m}{f_m}$$

3. Frequency deviation and bandwidth

Given:

$$k_f = 2 \text{ kHz/V}$$

Frequency deviation:

$$\Delta f = k_f A_m = 2 \times 2 = 4 \text{ kHz}$$

$$B_{FM} = 2(\Delta f + f_m)$$

$$B_{FM} = 2(4 + 1) = 10 \text{ kHz}$$

Exercise 3 :

Given

$$f = 2 \text{ kHz}$$

1. Sampling period

According to Nyquist:

$$f_e \geq 2f$$

$$f_e \geq 2 \times 2000 = 4000 \text{ Hz}$$

$$T_e = \frac{1}{f_e}$$

$$T_e = \frac{1}{4000}$$

$$T_e = 2.5 \times 10^{-4} \text{ s} = 0.25 \text{ ms}$$

$$T_e = 0.25 \text{ ms}$$

2. Sampling process (simplification)

- Sampling: convert $x(t)$ into discrete signal $x[n] = x(nT_e)$
- Quantization: map amplitudes to 2-bit levels (4 levels)
- Encoding: convert each level into binary words

3. Binary sequence

2-bit quantization \Rightarrow 4 levels

Example encoding:

$$0 \rightarrow 00, \quad 1 \rightarrow 01, \quad 2 \rightarrow 10, \quad 3 \rightarrow 11$$

$$x[n] \Rightarrow \text{binary sequence} : (b_1 b_2)(b_1 b_2) \dots$$

(depends on the sampled values of the given signal)

4. Manchester coding

- Bit 1: High \rightarrow Low
- Bit 0: Low \rightarrow High