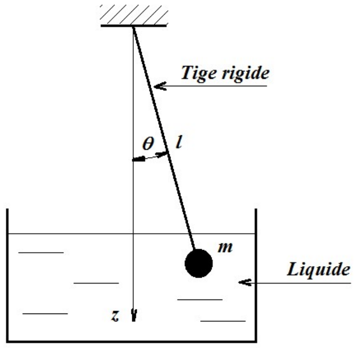
Corrigé Type

Exercice N°1 (10pts)



On donne $m = 1Kg$, $l = 80cm$ et $c = 1.85\frac{N-s}{m}$

1) Equation différentielle

2e Principe fondamental de la dynamique : $\sum M = J\ddot{\theta}$

$$-mglsin\theta - cl\dot{\theta} = ml^2\ddot{\theta} \quad \text{(1pt)}$$

En admettant que θ est petit :

$$ml^2\ddot{\theta} + cl\dot{\theta} + mgl\theta = 0$$

$$\ddot{\theta} + \frac{c}{ml}\dot{\theta} + \frac{g}{l}\theta = 0 \quad (1) \quad \text{(1pt)}$$

2-1) Résolution de l'équation différentielle (1)

Soient : $\omega^2 = \frac{g}{l}$ **(0.5pt)** et $q = \frac{c}{ml} = 2.31$

L'équation caractéristique :

$$s^2 + qs + \omega^2 = 0 \quad \text{(0.5pt)}$$

$$\Delta = q^2 - 4\omega^2 = \left(\frac{c}{ml}\right)^2 - 4\frac{g}{l} = \left(\frac{1.85}{1*0.8}\right)^2 - 4\frac{10}{0.8} = -44.65 = 44.65i^2 \quad \text{(0.5pt)}$$

Les solutions de l'eqt caractéristique :

$$s_1 = \frac{-2.31+6.68}{2} = -1.155 + 3.34i \quad \text{(0.5pt)}$$

$$s_2 = \frac{-2.31-6.68i}{2} = -1.155 - 3.34i \quad \text{(0.5pt)}$$

La solution de l'équation (1) est de la forme :

$$\theta(t) = A_1e^{(-1.155+3.34i)t} + A_2e^{(-1.155-3.34i)t} = e^{-1.155t}\left[A_1e^{3.34it} + A_2e^{-3.34it}\right] \quad \text{(1pt)}$$

$$\theta(t) = e^{-1.155t}[Acos(3.34t) + Bsin(3.34t)] \quad \text{(1pt)}$$

2-2) Détermination des constantes A et B

A t=0 $\theta(0) = 20° = \frac{\pi}{9}$ $\quad A = \frac{\pi}{9} = 0.348$ **(1pt)**

$\dot{\theta}(t) = -1.155 * e^{-1.155t}[Acos(3.34t) + Bsin(3.34t)] + e^{-1.155t}[-3.34Asin(3.34t) + 3.34Bcos(3.34t)]$ **(0.5pt)**

A t=0 $\dot{\theta}(0) = 0$ $\quad \dot{\theta}(0) = -1.155A + 3.34B = 0$ **(0.5pt)**

$B = \frac{1.155A}{3.34} = \frac{1.155*0.348}{3.34} = 0.12$ $\quad B = 0.12$ **(1pt)**

La solution définitive de l'équation différentielle :

$$\theta(t) = e^{-1.155t}[0.348\ cos(3.34t) + 0.12sin3.34t] \quad \text{(0.5pt)}$$

Exercice N°2

1) Equations différentielles

$$\begin{cases} -m\ddot{x}_1 - c(\dot{x}_1 - \dot{x}_2) - k(x_1 - x_2) + P(t) = 0 \\ -m\ddot{x}_2 - c(\dot{x}_2 - \dot{x}_1) - k(x_2 - x_1) - k(x_2 - x_3) = 0 \\ -m\ddot{x}_3 - k(x_3 - x_2) = P(t) \end{cases} \quad \text{(0.5*3=1.5pt)}$$

$$\begin{cases} m\ddot{x}_1 + c\dot{x}_1 - c\dot{x}_2 + kx_1 - kx_2 = P(t) \\ m\ddot{x}_2 - c\dot{x}_1 + c\dot{x}_2 - kx_1 + 2kx_2 - kx_3 = 0 \\ m\ddot{x}_3 - kx_2 + kx_3 = P(t) \end{cases} \quad \text{(0.25*3=0.75pt)}$$

2) Fréquences propres

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}\begin{bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \\ \ddot{z}_3 \end{bmatrix} + \begin{bmatrix} c & -c & 0 \\ -c & c & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} + \begin{bmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} P(t) \\ 0 \\ P(t) \end{bmatrix} \quad \text{(0.5*3=1.5pt)}$$

$$\begin{vmatrix} k-m\omega^2 & -k & 0 \\ -k & 2k-m\omega^2 & -k \\ 0 & -k & k-m\omega^2 \end{vmatrix} = 0$$ **(0.75pt)**

$$(k-m\omega^2)[(2k-m\omega^2)(k-m\omega^2)-k^2]-k^2(k-m\omega^2)=0$$ **(0.75pt)**

$$(k-m\omega^2)[2k^2-2km\omega^2-km\omega^2+m^2\omega^2+k^2]-k^2(k-m\omega^2)=0$$

$$(k-m\omega^2)[3k^2-3km\omega^2+m^2\omega^4]-k^2(k-m\omega^2)=0$$

$$(k-m\omega^2)[m^2\omega^4-3km\omega^2+2k^2]=0$$ **(0.75pt)**

(0.25pt) $k-m\omega^2=0$ --> $\omega_0^2=\frac{k}{m}=1000$ ---> $\omega_0=31.62rds/s$ **(0.5pt)**

(0.25pt) $m^2\omega^4-3km\omega^2+2k^2=0$ -->$\Delta= 9m^2k^2-4*m^2*2k^2=m^2k^2$

(0.25pt) $\omega_1^2=\frac{3km+km}{2*m^2}=2000$ ---> $\omega_1=44.72rds/s$ **(0.5pt)**

(0.25pt) $\omega_2^2=\frac{3km-km}{2*m^2}=1000$ ---> $\omega_2=31.62rds/s$ **(0.5pt)**

3-1) Taux d'amortissement de m_1

(0.25pt) $\xi=\frac{c}{c_c}=\frac{c}{2m\omega}=\frac{c}{2m\sqrt{\frac{k}{m}}}=1.58$ **(0.5pt)**

3-2) Amplitude

(0.25pt) $U_0=\frac{P_0}{k}\frac{1}{\sqrt{(1-\beta^2)^2+(2\xi\beta)^2}}=0.5933m=59.33cm$ **(0.5pt)**