Exam correction

Exercise 1 (08 pts).

The heights X (in cm) of 100 students were measured. The results are presented in the following table:

X_i	$[150;160[$	$[160;165[$	$[165;170[$	$[170;175[$	$[175;180[$	$[180;190[$	$TOTAL$
n_i	8	24	42	14	10	2	$N=100$
c_i	155	162.5	167.5	172.5	177.5	185	/
f_i	0.08	0.24	0.42	0.14	0.10	0.02	1
F_i	0.08	0.32	0.74	0.88	0.98	1	/
$n_i c_i$	1240	3900	7053	2415	1775	370	16735
$n_i c_i^2$	192200	633750	1178362.5	416587.5	315062.5	68450	2804412.5

(1) $\star$ $\bar{X} = \frac{1}{N}\sum_1^6 n_i c_i = 167.35$

$\star$ $Var(X) = \frac{1}{N}\sum_1^6 n_i c_i^2 - \bar{X}^2 = 38.1025$

$\star$ $\sigma_X = \sqrt{Var(X)} = 6.173$

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(2) The median (Me) *using linear interpolation:*

$$\frac{0.5 - F(X_1)}{Me - X_1} \sim \frac{F(X_{i+1}) - F(X_1)}{X_{i+1} - X_1}$$

$$[(0.5 - F(X_1))(X_{i+1} - X_1)] \sim [(Me - X_1)(F(X_{i+1}) - F(X_1))]$$

$$\Rightarrow M_e \sim X_1 + (X_{i+1} - X_1)\frac{0.5 - F(X_1)}{F(X_{i+1}) - F(X_1)}$$

$[X_1; X_{1+i}[\rightarrow [165;170[,\ F(165) \leq 0.5 \leq F(170).$

$M_e \sim 165 + (170 - 165)\frac{0.5 - F(165)}{F(170) - F(165)} = 167.143.$

(3) $Z_1 = \bar{X} - \sigma_X = 167.35 - 6.173 = 161.177 \in [160;165[.$

$Z_2 = \bar{X} + \sigma_X = 167.35 + 6.173 = 173.523 \in [170;175[.$

$$\frac{F(Z) - F(X_1)}{F(X_2) - F(X_1)} \sim \frac{Z - X_1}{X_2 - X_1}$$

$F(z) \sim F(X_1) + [F(X_2) - F(X_1)]\frac{Z - X_1}{X_2 - X_1}.$

$F(Z_1) \sim 0.1365,\ F(Z_2) \sim 0.8386,\ [F(Z_2) - F(Z_1)] \times 100 = 70,21\%$

Exercise 2 (08 pts).

A pharmaceutical product has just been launched on the market. It has experienced considerable success during the first 8 months. The recorded sales are presented in the table below:

T	1	2	3	4	5	6	7	8	/
V	10	23	38	77	165	318	642	1270	/
$n_iT_i^2$	1	4	9	16	25	36	49	64	204
$n_iV_i^2$	100	529	1444	5929	27225	101124	412164	1612900	2161415
$T_iV_i^2$	10	46	114	308	825	1908	4494	10160	17865
$lnV_i = W_i$	2.3025	3.1354	3.6375	4.3438	5.1059	5.7620	6.4645	7.1467	37.8983
T_iW_i	2.3025	6.2709	10.9127	17.3752	25.5297	34.5723	45.2521	57.1741	199.3895

(1) ⋆ $\bar{T} = \frac{1}{N}\sum_{i=1}^{8} n_iT_i = 4.5.$

⋆ $Var(T) = \frac{1}{N}\sum_{i=1}^{8} n_iT_i^2 - \bar{T}^2 = 5.25.$

⋆ $\sigma_T = \sqrt{Var(T)} = 2.291.$

⋆ $\bar{V} = \frac{1}{N}\sum_{j=1}^{8} n_jV_j = 317.875.$

⋆ $Var(V) = \frac{1}{N}\sum_{j=1}^{8} n_jV_j^2 - \bar{V}^2 = 169132.3594.$

⋆ $\sigma_V = \sqrt{Var(V)} = 411.257.$

⋆ $Cov(T,V) = \frac{1}{N}\sum_{i,j=1}^{8} n_{ij}T_iV_j - \bar{T}\bar{V} = 802.6875.$

(2) $r = \dfrac{Cov(T,V)}{\sigma_T \times \sigma_V} = 0.8519.$

(3)

$$\begin{aligned} V &= BA^T, \\ lnV &= lnBA^T, \\ lnV &= TlnA + lnB. \\ \mathcal{V} &= \mathcal{T}a + b. \end{aligned}$$

$$\begin{cases} a &= lnA \\ b &= lnB \end{cases} \Rightarrow \begin{cases} A &= e^a \\ B &= e^b \end{cases} \text{ and } \begin{cases} \mathcal{V} &= lnV \\ \mathcal{T} &= T \end{cases}$$

$$\begin{cases} a &= \frac{cov(T,lnV)}{Var(T)} = 0.68 \\ b &= \bar{lnV} - a\bar{T} = 1.64 \end{cases} \Rightarrow \begin{cases} A &= e^{0.68} = 1.98. \\ B &= e^{1.64} = 5.18. \end{cases}$$

$$V = 5.18(1.98)^T$$

Exercise 3 (02 pts). $\Omega = \{0.1.2.3.4.5.6.7.8.9\}$, $\mathcal{Q}_{10}^6 = 10^6$.

Exercise 4 (02 pts).

(1) $Card(\Omega) = 6$, $A = \{2,3,6\}$, $B = \{3,6\}$.

⋆ $P(A) = \dfrac{Card(A)}{Card(\Omega)} = \frac{3}{6} = \frac{1}{2}.$

⋆ $P(B) = \dfrac{Card(B)}{Card(\Omega)} = \frac{2}{6} = \frac{1}{3}.$

(2) $\star$ $P(A \cap B) = \frac{1}{6}$.

$\star$ $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{2}$.

Good luck