

Final exam of Electricity

Questions: (1.25 pts)

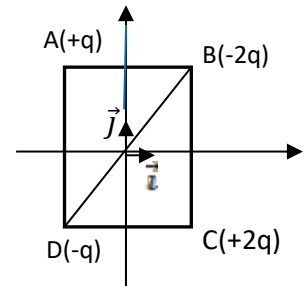
Choose the correct answer:

- Like charges: a- repel. b- attract each other.
- Electric force is: a- $F = k \frac{|q_1 \cdot q_2|}{r}$ b- $F = k \frac{|q_1 \cdot q_2|}{r^2}$ c- $F = k \frac{|q_1 \cdot q_2|}{\sqrt{r}}$
- The electric field is zero: a - inside the conductor. b- outside a charged conductor. c- just outside a charged conductor.

Exercise 1: (7.25pts)

Four electric point charges $q_A = q$, $q_B = -2q$, $q_C = 2q$ and $q_D = -q$ are placed at the vertices A, B, C and D of a square with side length a.

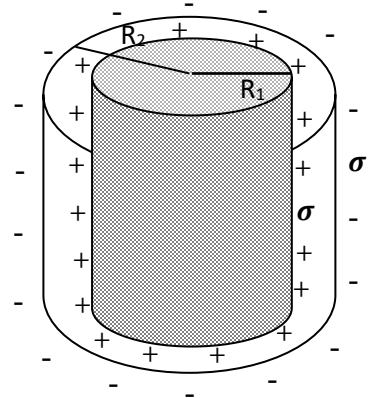
- Represent and find the electrostatic field at the center O of the square.
- We put a charge $Q = -q$ at point O, represent and find the electrostatic force exerted on this charge.
- Determine the electric potential V(O) at point O.
- What is the value of the potential energy at this point $E_P(O)$.



Exercise 2: (5.25pts)

Consider two cylinders in a vacuum with the same axis, with radii R_1 and R_2 where $R_1 < R_2$, of negligible thickness, and uniformly charged on their surfaces, with charge densities $+\sigma$ and $-\sigma$, respectively.

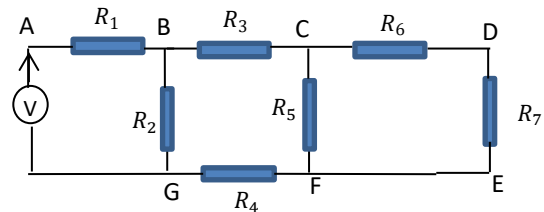
- Determine the electric field at all points in space.
- Calculate the difference in potential ΔV between the two cylinders.



Exercise 3: (6,25 pts)

Let the circuit in the following figure

- Name the circuit elements: number of nodes, number of branches and number of loops.
- Determine the equivalent circuit resistance and give the simplified circuit.
- Calculate the current intensity I.



$V=20 \text{ V}, R_1 = R_3 = R_5 = 100 \Omega, R_2 = 200 \Omega$ and $R_4 = R_6 = R_7 = 50 \Omega$.

Good Luck.

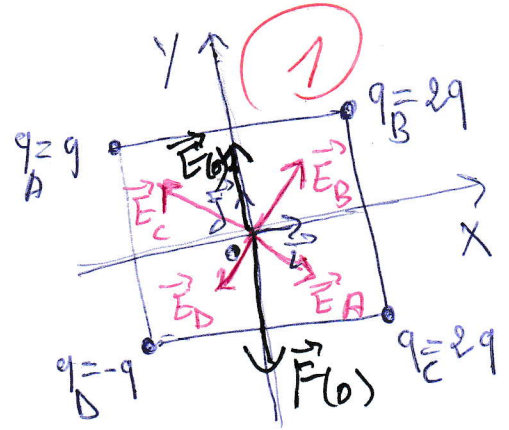
Model answer for Final Exam 1- 1st C.S Electricity 2025-2026

Questions: (4 pts)

1. → a (0.25), 2. → b (0.15), 3. → a. (0.15)

EX n° 1:

1- Electric field \vec{E} at center O:



we have: $\vec{E}(O) = \vec{E}_A + \vec{E}_B + \vec{E}_C + \vec{E}_D$ (0.15)

By reason of symmetry, we have:

$$\vec{E}_A = \vec{E}_D \Rightarrow \vec{E}_A + \vec{E}_D = -2E_A \cos \frac{\pi}{4} \vec{j} = \frac{-2kq}{\|\vec{AO}\|^2} \cos \frac{\pi}{4} \vec{j} \quad (0.25)$$

In triangle ADC, we have: $a^2 + a^2 = AC^2 \Rightarrow AC = \sqrt{2} a$ (0.25)

$$\Rightarrow \|\vec{AO}\| = \frac{AC}{2} = \frac{\sqrt{2}}{2} a \quad (0.25)$$

$$\text{Hence: } \vec{E}_A + \vec{E}_D = \frac{-2k(q)}{\left(\frac{\sqrt{2}}{2}a\right)^2} \cdot \frac{\sqrt{2}}{2} \vec{j} = \frac{-2\sqrt{2}kq}{a^2} \vec{j} \quad (0.25)$$

we have the same: $\vec{E}_C = \vec{E}_B \Rightarrow \vec{E}_C + \vec{E}_B = 2E_C \cos \frac{\pi}{4} \vec{j}$ (0.25)

$$\Rightarrow \vec{E}_C = \frac{2kq_c}{\|\vec{CO}\|^2} \cos \frac{\pi}{4} \vec{j} = \frac{2k(2q)}{\left(\frac{\sqrt{2}}{2}a\right)^2} \cdot \frac{\sqrt{2}}{2} \vec{j} = \frac{4kq\sqrt{2}}{a^2} \vec{j} \quad (0.25)$$

$$\text{So, } \vec{E}(O) = \frac{-2kq\sqrt{2}}{a^2} \vec{j} + \frac{4kq\sqrt{2}}{a^2} \vec{j} \Rightarrow \vec{E}(O) = \frac{2kq\sqrt{2}}{a^2} \vec{j} \quad (0.25)$$

$$2/ \vec{F}(O) = q\vec{E}(O) = \frac{kq^2}{a^2} \vec{j} \quad (0.15)$$

$$3/ V(O) = V_A + V_B + V_C + V_D = \frac{kq_A}{AO} + \frac{kq_B}{BO} + \frac{kq_C}{CO} + \frac{kq_D}{DO} \quad (0.15)$$

$$V(0) = K \left(\frac{q}{a} + \frac{-2q}{a} + \frac{2q}{a} + \frac{-q}{a} \right) = 0 \quad (0.15)$$

$$4/ \quad E_p(0) = q \left(\frac{1}{a} - \frac{2}{a} + \frac{2}{a} - \frac{1}{a} \right) = -q \times 0 = 0 \quad (0.15)$$

Ex n° 2

1/ By consideration of symmetry $\vec{E}(r)$ is radial

$$\vec{E}(r) = E(r) \vec{u}_r \quad (0.15)$$

$$\oiint \vec{E} \cdot d\vec{s} = \frac{\Sigma Q_{int}}{\epsilon_0} \Rightarrow E \cdot S_G = \frac{\Sigma Q_{int}}{\epsilon_0} \quad (0.25)$$

we have 3 cases:

* 1st case: $R < R_1$ (0.25)

$$Q_{int} = 0 \Rightarrow E_1(r) = 0 \quad (0.15)$$

* 2nd case: $R_1 < r < R_2$ (0.25)

$$Q_{int} = \sigma S = 2\pi R_1 h \sigma \quad (0.15), \quad S_G = 2\pi R_1 h$$

$$E_2 \cdot S_G = \frac{Q_{int}}{\epsilon_0} \Rightarrow E_2 \cdot 2\pi r h = \frac{2\pi R_1 h \sigma}{\epsilon_0} \Rightarrow E_2(r) = \frac{\sigma R_1}{\epsilon_0 r} \quad (0.25)$$

* 3rd case: $r > R_2$ (0.25)

$$Q_{int} = \sigma 2\pi R_1 h - \sigma 2\pi R_2 h = 2\pi \sigma (R_1 - R_2) \quad (0.15), \quad S_G = 2\pi r h$$

$$E_3 \cdot S_G = \frac{Q_{int}}{\epsilon_0} \Rightarrow E_3 \cdot 2\pi r h = \frac{2\pi \sigma (R_1 - R_2)}{\epsilon_0} \Rightarrow E_3 = \frac{\sigma}{\epsilon_0} \left(\frac{R_2 - R_1}{r} \right) \quad (0.25)$$

2/ $\Delta V = ?$ (0.25)

$$\vec{E} = -\text{grad } V = -\int_{V(R_1)}^{R_2} \vec{E} \cdot d\vec{r}$$

$$\Delta V = V(R_2) - V(R_1) = -\int_{R_1}^{R_2} \frac{\sigma R_1}{\epsilon_0 r} dr = -\frac{\sigma R_1}{\epsilon_0} \ln r \Big|_{R_1}^{R_2} \Rightarrow \Delta V = \frac{\sigma R_1}{\epsilon_0} \ln \frac{R_1}{R_2} \quad (0.15)$$

EX n° 3.

1/ The elements of the circuit:

we have:

* 4 nodes: B, C, F and G. (1)

* 6 branches: AB, BC, CDEF, FG, GB and CF. (1.15)

* 3 loops: ABGA, BCFG and CDEF. (0.175)

2/ The equivalent resistance:

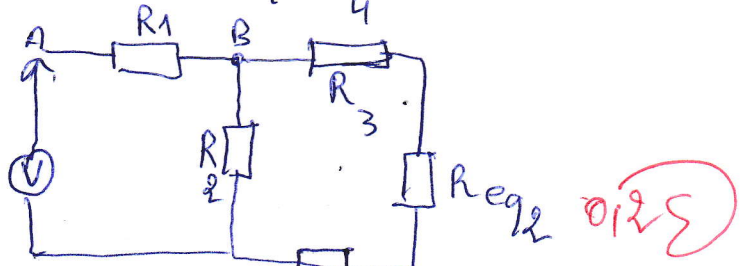
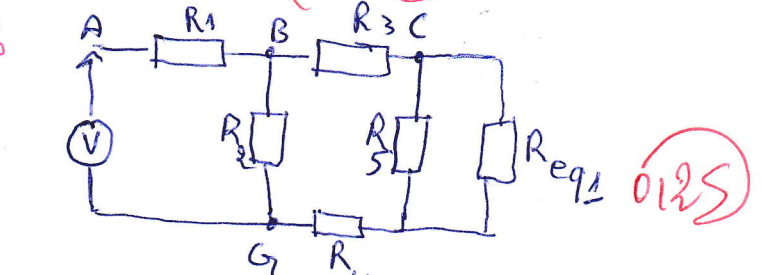
* R_6 and R_7 in serie \Rightarrow

$$R_{eq} = R_6 + R_7 = 50 + 50 = 100 \Omega$$

* $R_{eq1} \parallel R_5 \Rightarrow$

$$R_{eq} = \left(\frac{1}{R_{eq1}} + \frac{1}{R_5} \right)^{-1} = \left(\frac{1}{100} + \frac{1}{100} \right)^{-1} = \frac{2}{10}$$

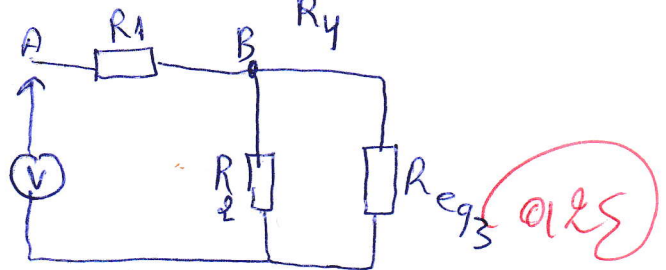
$$\Rightarrow R_{eq2} = 50 \Omega$$



* R_{eq2} , R_3 and R_4 in serie \Rightarrow

$$R_{eq3} = R_{eq2} + R_3 + R_4 = 50 + 50 + 100$$

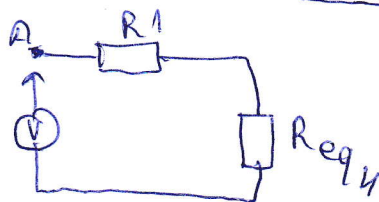
$$R_{eq3} = 200 \Omega$$



* $R_{eq3} \parallel R_2 \Rightarrow$

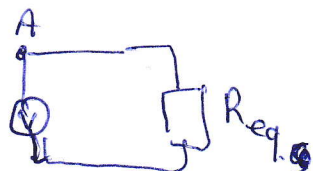
$$\frac{1}{R_{eq4}} = \left(\frac{1}{R_{eq3}} + \frac{1}{R_2} \right)^{-1} = \left(\frac{1}{200} + \frac{1}{200} \right)^{-1} = \frac{2}{200}$$

$$\Rightarrow R_{eq4} = 100 \Omega$$



* R_{eq4} and R_1 in serie $\Rightarrow R_{eq} = R_{eq4} + R_1 = 100 + 100$

$$\Rightarrow R_{eq} = 200 \Omega$$



3/ The current intensity I:

$$V = R_{eq} I \Rightarrow I = \frac{V}{R_{eq}} = \frac{20}{200} = 0.1 \text{ A}$$