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L3-SI | Year 2025/2026
Linear programming | Enseignant : A.Khiar

Final Exam Correction

Questions (3 pts)

1st R: If we have (P) in Standard Form $P = \{x \in \mathbb{R}^n : Ax = b, x \geq 0\}$, where $A \in \mathbb{R}^{m\times n}$ has rank m

x* Basic Feasible Solution (BFS) :

Feasible: $A x^* = b, x \geq 0$

Basic: There exist m linearly independent columns of A where: **1.5**

$A(m)\, x^* = b$

2nd R : - Two extreme points are adjacent, if it differs from a single hyperplane.

- two BFSs are adjacent, if it differs by a single column. **1.5**

Exercise 1 (7 pts)

1. Definition of decision variables

To maximize the profits of the company we must increase the quantities of sales that bring benefits to the company.

x_1 : quantity sold of products type P1
x_2 : quantity sold of products type P2 **1**
x_3 : quantity sold of products type P3

Concerning the 4th product, the company defines packaging according to the production of P1, P2, P3

x4: quantity sold of products type P4

2. Objective function

$$\max\ z = f(x_1, x_2, x_3) = 14x_1 + 12x_2 + 16x_3$$ **1**

3. Constraints:
   a. Limit of stock in M1: Qte_P1+Qte_P2+Qte_P3+Qte_P4 <= 600 **0.5**

   $$2x_1 + 2x_2 + 2x_3 + 1x_4 \leq 600$$

   b. Limit of stock in M2: Qte_P1+Qte_P2+Qte_P3+Qte_P4 <= 800 **0.5**

   $$3x_1 + 1x_2 + 2x_3 + 1x_4 \leq 800$$

   c. Time Constraint

   Let T1,T2,T3,T4 be the unit times to produce P1,P2,P3,P4

   T2 = 1200/500 = 12/5 and T1+T3+T4 = 3 * T2 = 36/5

   Soit :

   $$T1 * x_1 + \frac{12}{5} * x_2 + T3 * x_3 + T4 * x_4 \leq 1200$$ **0.5**

   $$T1 + T3 + T4 = \frac{36}{5}$$ **0.5**

   d. Product Demand:
   - **P1 :** $x_1 \geq 200$
   - **P2 :** $x_2 \geq 150$ $x_2 \leq 300$ **1**
   - **P3 :** $x_3 \geq 250$

e. Free production of P4 :

0.5

$$\mathbf{10 * (x_1 + x_2 + x_3) = x_4}$$

f. Positivity constraints

$$\mathbf{x_1, x_2, x_3, x_4, T1, T3, T4 \geq 0}$$

0.5

Let (P) be the resulting program

$$(P)\begin{cases} \max z = f(x_1,x_2,x_3) = 14x_1 + 12x_2 + 16x_3 \\ s.c \begin{cases} 2x_1 + 2x_2 + 2x_3 + 1x_4 \leq 600 \\ 3x_1 + 1x_2 + 2x_3 + 1x_4 \leq 800 \\ T1 * x_1 + \frac{12}{5} * x_2 + T3 * x_3 + T4 * x_4 \leq 1200 \\ T1 + T3 + T4 = \frac{36}{5} \\ 10 * (x_1 + x_2 + x_3) = x_4 \\ x_1 \geq 200 \\ x_2 \geq 150 \\ x_2 \leq 300 \\ x_3 \geq 250 \\ x_1, x_2, x_3, x_4, T1, T3, T4 \geq 0 \end{cases} \end{cases}$$

1

Exercise 2 : (10 pts)

1. Canonical form of (P)

Pb of maximization then all the constraints will be of the **<=** form

$\mathbf{2x_1 + x_2 + x_3 \geq 25} \rightarrow \mathbf{-2x_1 - x_2 - x_3 \leq -25}$

$\mathbf{1x_1 + 2x_2 + 3x_3 = 45} \rightarrow \begin{cases} \mathbf{1x_1 + 2x_2 + 3x_3} \geq 45 \rightarrow \mathbf{-1x_1 - 2x_2 - 3x_3} \leq -45 \\ \mathbf{1x_1 + 2x_2 + 3x_3} \leq 45 \end{cases}$

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$$(P)\begin{cases} \max z = f(x_1,x_2,x_3) = 3x_1 + x_2 - 2x_3 \\ s.c \begin{cases} \mathbf{-2x_1 - x_2 - x_3 \leq -25} \\ \mathbf{-1x_1 - 2x_2 - 3x_3} \leq -45 \\ \mathbf{1x_1 + 2x_2 + 3x_3} \leq 45 \\ \mathbf{x_1 + 3x_2 + x_3 \leq 50} \\ x_1, x_2, x_3 \geq 0 \end{cases} \end{cases}$$

2. Standard form of (P)

Add the variance variables in the constraints of type <= and >=

$$(P)\begin{cases} \mathbf{\max z = f(x_1,x_2,x_3) = 5x_1 + 4x_2 + 4.5x_3} \\ \textbf{\textit{Subject}} \begin{cases} \mathbf{2x_1 + x_2 + x_3 - e_1 = 25} \\ \mathbf{1x_1 + 2x_2 + 3x_3 = 45} \\ \mathbf{x_1 + 3x_2 + x_3 + e_2 = 50} \\ \\ \mathbf{x_1, x_2, x_3 \geq 0} \end{cases} \end{cases}$$

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3. Solving (P) by the simplex method

Matrix form of (P)

$$\begin{pmatrix} 2 & 1 & 1 & -1 & 0 \\ 1 & 2 & 3 & 0 & 0 \\ 1 & 3 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} 25 \\ 45 \\ 50 \end{pmatrix}$$

Note that there is no obvious solution for this problem, → need phase I to define an initial solution

Phase I: Finding an initial feasible basic solution

Two artificial variables defined on the first two constraints to have a basis

Let (PA) be the artificial program for (P)

$$(P)\begin{cases} \min q = a_1 + a_2 \\ \textit{Subject} \begin{cases} 2x_1 + x_2 + x_3 - e_1 + a_1 = 25 \\ 1x_1 + 2x_2 + 3x_3 + a_2 = 45 \\ x_1 + 3x_2 + x_3 + e_2 = 50 \\ \\ x_1, x_2, x_3 \geq 0 \end{cases} \end{cases}$$

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$$\begin{pmatrix} 2 & 1 & 1 & -1 & 0 & 1 & 0 \\ 1 & 2 & 3 & 0 & 0 & 0 & 1 \\ 1 & 3 & 1 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ e_1 \\ e_2 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 25 \\ 45 \\ 50 \end{pmatrix}$$

(PA) accepts (a1=25; a2=45; e2=50) as the initial BFS.

Phase I : Table 1 :

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min	ci	0	0	0	0	0	1	1			
CB	B	b/v	x1	x2	x3	e1	e3	a1	a2	Ratio	B (a1,a2,e2) = (25,45,50) V.E : x3 V.S : a2
1	a1	25	2	1	1	-1	0	1	0	25	
1	a2	45	1	2	3	0	0	0	1	15	
0	e3	50	1	3	1	0	1	0	0	50	
	Q	70	3	3	4	-1	0	1	1		
	E		-3	-3	-4	1	0	0	0		

Table 2 :

min	ci		0	0	0	0	0	1		
CB	B	b/v	x1	x2	x3	e1	e3	a1	ratio	
1	a1	10	1 2/3	1/3	0	-1	0	1	6	
0	x3	15	1/3	2/3	1	0	0	0	45	B (a1,x3,e2) = (10,15,35) V.E : x1 V.S : a1
0	e3	35	2/3	2 1/3	0	0	1	0	52 1/2	
	Q	10	1 2/3	1/3	0	-1	0	1		
	E		-1 2/3	- 1/3	0	1	0	0		

Table 3 :

min	ci	0	0	0	0	0		
CB	B	b/v	x1	x2	x3	e1	e3	Q= 0 → arret de phase I Inital BFS for (P) B =(x1,x3,e2) = (6,13,31)
0	x1	6	1	1/5	0	- 3/5	0	
0	x3	13	0	3/5	1	1/5	0	
0	e2	31	0	2 1/5	0	2/5	1	
	Q	0	0	0	0	0	0	
	E		0	0	0	0	0	

Phase II : Table 1 :

Max	ci		5	4	4 1/2	0	0	
CB	B	b/v	x1	x2	x3	e1	e2	ration
5	x1	6	1	1/5	0	- 3/5	0	-10
4 1/2	x3	13	0	3/5	1	1/5	0	65
0	e2	31	0	2 1/5	0	2/5	1	77 1/2
	z	88 1/2	5	3 7/10	4 1/2	-2 1/10	0	
	E		0	3/10	0	2 1/10	0	

B =(x1,x3,e2) = (6,13,31)
V.E : e1
V.S : x3

Table 2 :

Max	ci		5	4	4 1/2	0	0	
CB	B	b/v	x1	x2	x3	e1	e2	ratio
5	x1	45	1	2	3	0	0	
0	e1	65	0	3	5	1	0	
0	e2	5	0	1	-2	0	1	
	z	225	5	10	15	0	0	
	E		0	-6	-10 1/2	0	0	

EN < 0
Algorithem stopped
final BFS is
B =(x1,e1,e2)=(45,64,5)

Optimal solution
X1=45,x2=0,x3=0
Z = 225

4. Dual Program of (P)

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From general form

$$(P)\begin{cases} \max z = f(x_1,x_2,x_3) = 5x_1 + 4x_2 + 4.5x_3 \\ \textit{Subject} \begin{cases} 2x_1 + x_2 + x_3 \geq 25 \\ 1x_1 + 2x_2 + 3x_3 = 45 \\ x_1 + 3x_2 + x_3 \leq 50 \\ x_1, x_2, x_3 \geq 0 \end{cases} \end{cases}$$

max → min
3 contraintes → 3 variables
3 variables → 3 contraintes
c → b
b→ c
………

$$(DP)\begin{cases} \min d = g(y_1,y_2,y_3) = 25y_1 + 45y_2 + 50y_3 \\ subject \begin{cases} 2y_1 + y_2 + y_3 \geq 5 \\ y_1 + 2y_2 + 3y_3 \geq 4 \\ y_1 + 3y_2 + y_3 \geq 4.5 \\ y_1 \leq 0 \ , y_2 \in \mathbb{R}, y_3 \geq 0 \end{cases} \end{cases}$$

All that remains is to normalize y1 and y2

From canonical form

$$(P)\begin{cases} \max z = f(x_1,x_2,x_3) = 5x_1 + 4x_2 + 4.5x_3 \\ subject \begin{cases} -2x_1 - x_2 - x_3 \leq -25 \\ -1x_1 - 2x_2 - 3x_3 \leq -45 \\ 1x_1 + 2x_2 + 3x_3 \leq 45 \\ x_1 + 3x_2 + x_3 \leq 50 \\ x_1, x_2, x_3 \geq 0 \end{cases} \end{cases}$$

max → min
4 contraintes → 4 variables
3 variables → 3 contraintes

$$(DP)\begin{cases} \min d = g(y_1,y_2,y_3) = -25y_1 - 45y_2 + 45y_3 + 50y_4 \\ subject \begin{cases} -2y_1 - y_2 + y_3 + y_4 \geq 5 \\ -y_1 - 2y_2 + 2y_3 + 3y_4 \geq 4 \\ -y_1 - 3y_2 + 3y_3 + y_4 \geq 4.5 \\ y_1 \geq 0 \ , y_2 \geq 0, y_3 \geq 0, y_4 \geq 0 \end{cases} \end{cases}$$

5. Deduction of the solution of (PD)

d = z = 225 → $-25y_1 - 45y_2 + 45y_3 + 50y_4 = 225$

x1 > 0 → first constraint in (PD) is saturated → $-2y_1 - y_2 + y_3 + y_4 = 5$

x2 = 0 → $-y_1 - 2y_2 + 2y_3 + 3y_4 > 4$

x3 = 0 → $-y_1 - 3y_2 + 3y_3 + y_4 > 4.5$

we have this system :

$$subject \begin{cases} -25y_1 - 45y_2 + 45y_3 + 50y_4 = 225 \\ -2y_1 - y_2 + y_3 + y_4 = 5 \\ -y_1 - 2y_2 + 2y_3 + 3y_4 > 4 \\ -y_1 - 3y_2 + 3y_3 + y_4 > 4.5 \\ y_1 \geq 0 \ , y_2 \geq 0, y_3 \geq 0, y_4 \geq 0 \end{cases}$$

We done after resolution :

Y1 =0, y2 = 0, y3 =5, y4=0

D = 225