University Abbes Laghrour Khenchela
Faculty of Science and Technology
Department of Electrical Engineering

3rd Year in Telecommunications
Academic Year: 2025/2026

Final Exam -Antennas and Transmission Lines-

Duration: 01 hour 30 minutes

First Name:

Last Name:

Formula reminder:

Conversion from Cartesian to Spherical coordinates:

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Differential surface element in spherical coordinates:

$$\overrightarrow{dS} = r^2 \sin(\theta)\, d\theta d\varphi \vec{a}_r + r\sin(\theta)\, dr d\varphi \vec{a}_\theta + r dr d\theta \vec{a}_\varphi$$

Course Questions (4 points) :

1) List the different types of radiation regions ....
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2) Give the definition of an antenna ....
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3) Draw the equivalent circuit of an antenna ....
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4) Give the equivalent circuit of a transmission line section of length Δz ....
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Exercise 1 (6 points):

A radio-frequency source operating at a frequency of $5MHz$ feeds an antenna through a lossless transmission line. The antenna input impedance is $Z_L = (150 + j50)\Omega$. The transmission line is a coaxial cable of length l=15m, characteristic impedance $Z_0 = 50\Omega$, and propagation velocity v= $2\times10^8 m/s$.

Determine:

1) The reflection coefficient Γ_L at the load.

2) The standing wave ratio SWR.

3) The input impedance Z_{in}.

4) Verify the previous results using the Smith chart.

Exercise 2 (7 points):

The figure below represents three Hertzian dipoles **A**, **B**, and **C** placed in a Cartesian coordinate system. Each dipole has an infinitesimal length *dl* and carries a current $I = I_0 e^{j\omega t}$.

For the Hertzian dipole **A**, determine:

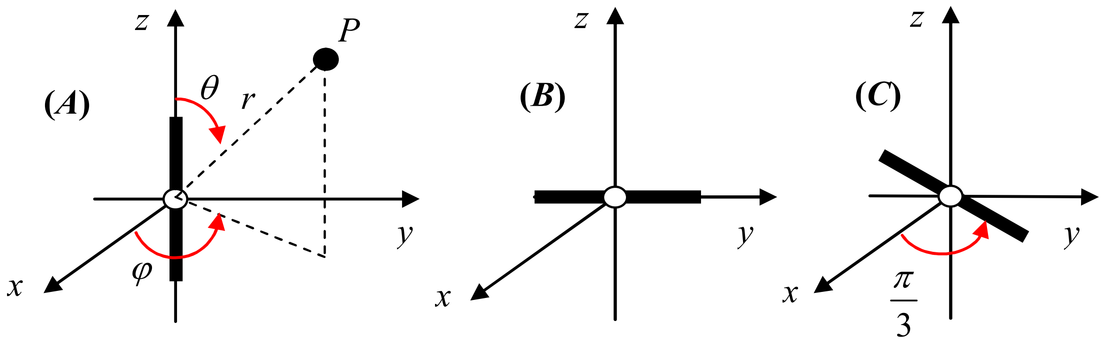
1) The expressions of the electric field **E** and magnetic field **H** radiated at a point **P** in the far-field region.

2) The expression of the radiated power density $\vec{\wp}_{ave}$.

3) The expression of the total radiated power P_r aver all space, and the radiation resistance R_r.

4) The directivity $D(\theta,\varphi)$ and the maximum directivity $D_{\max}$.

5) Considering the two Hertzian dipoles **B** and **C**, determine in the far-field region: the expressions of the electric and magnetic fields radiated by the two dipoles.



Exercise 3 (3 points):

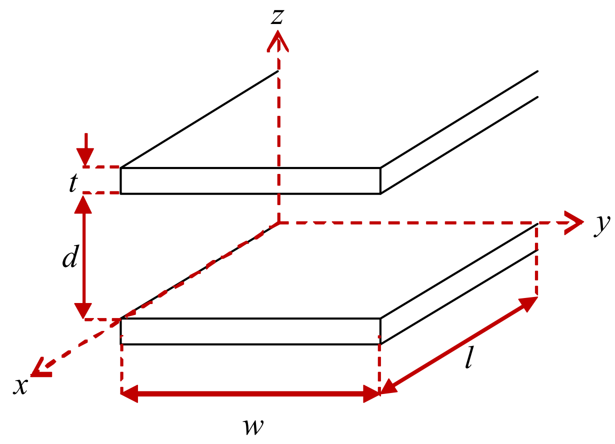
Consider two identical parallel conducting plates separated by a homogeneous dielectric medium by a distance *d*. Each conducting plate has a thickness *t*.

At high frequency, determine the per-unit-length resistance R_l and the per-unit-length capacitance C_l.

Given:

-Each conducting plate is characterized by its electrical conductivity σ_c, permeability μ_c, and permittivity ε_c.

-The dielectric medium separating the plates is characterized by its conductivity σ, permeability μ, and permittivity ε.



Good Luck !

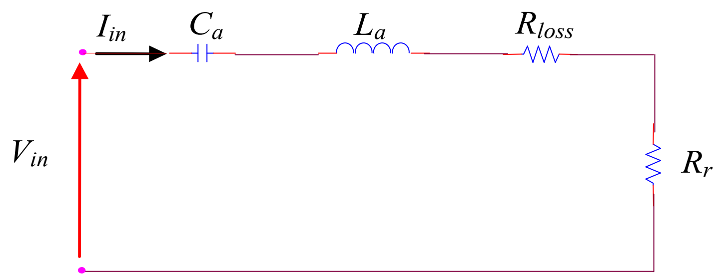
Correction of the Final Exam: Antennas and Transmission Lines

Course Questions (4 points):

1) The antenna radiation regions are typically divided into three zones: the Rayleigh zone (reactive near-field region), the Fresnel zone (radiating near-field region), and the Fraunhofer zone (far-field region). 1pt

2) Definition of an antenna: An antenna is a device that provides coupling between a guided electromagnetic wave propagating along a transmission line and a wave radiated into free space. In other words, antennas act as an interface between the transmitter/receiver system and the surrounding environment. 1pt

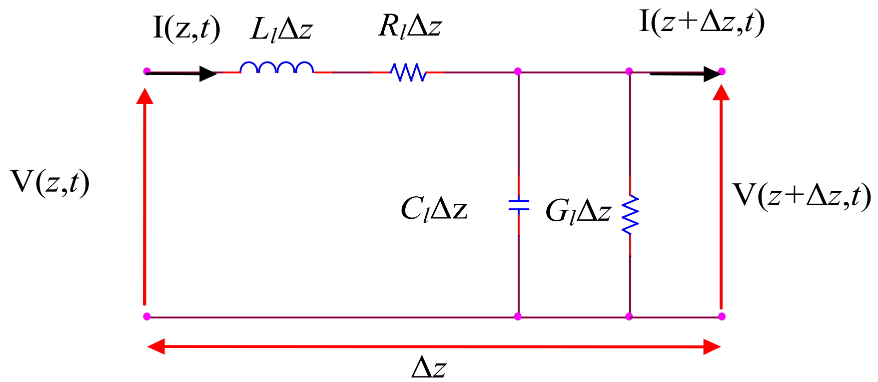
3) Equivalent circuit of an antenna:



0.5pt

The equivalent circuit of an antenna is generally represented by a radiation resistance R_r, a loss resistance R_{loss}, and a reactive component X_A (which may be capacitive C_a, or inductive L_a). 0.5pt

4) Equivalent circuit of a transmission line section of length Δz :



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The four primary transmission-line parameters R_l, L_l, C_l, and G_l are defined per unit length and have the following units: 0.5pt

R_l: Resistance per unit length (Ω/m), L_l: Inductance per unit length (H/m), C_l: Capacitance per unit length (F/m), G_l: Conductance per unit length (S/m)

Exercise 1 (6 points):

1) Reflection coefficient:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{150 + j50 - 50}{150 + j50 + 50} = \frac{100 + j50}{200 + j50} = \frac{2 + j}{4 + j} \approx 0.53 + j0.12$$ **1pt**

Magnitude: $|\Gamma_L| = \sqrt{0.53^2 + 0.12^2} \approx 0.54$ **0.25pt**

Phase: $\Phi_{\Gamma_L} = \tan^{-1}\left(\frac{\text{Im}}{\text{Re}}\right) = \tan^{-1}\left(\frac{0.12}{0.53}\right) \approx 12.76^0$ **0.25pt**

Thus: $\Gamma_L = |\Gamma_L| e^{j\Phi_{\Gamma_L}} \approx 0.54 e^{j12.76^0}$ **0.5pt**

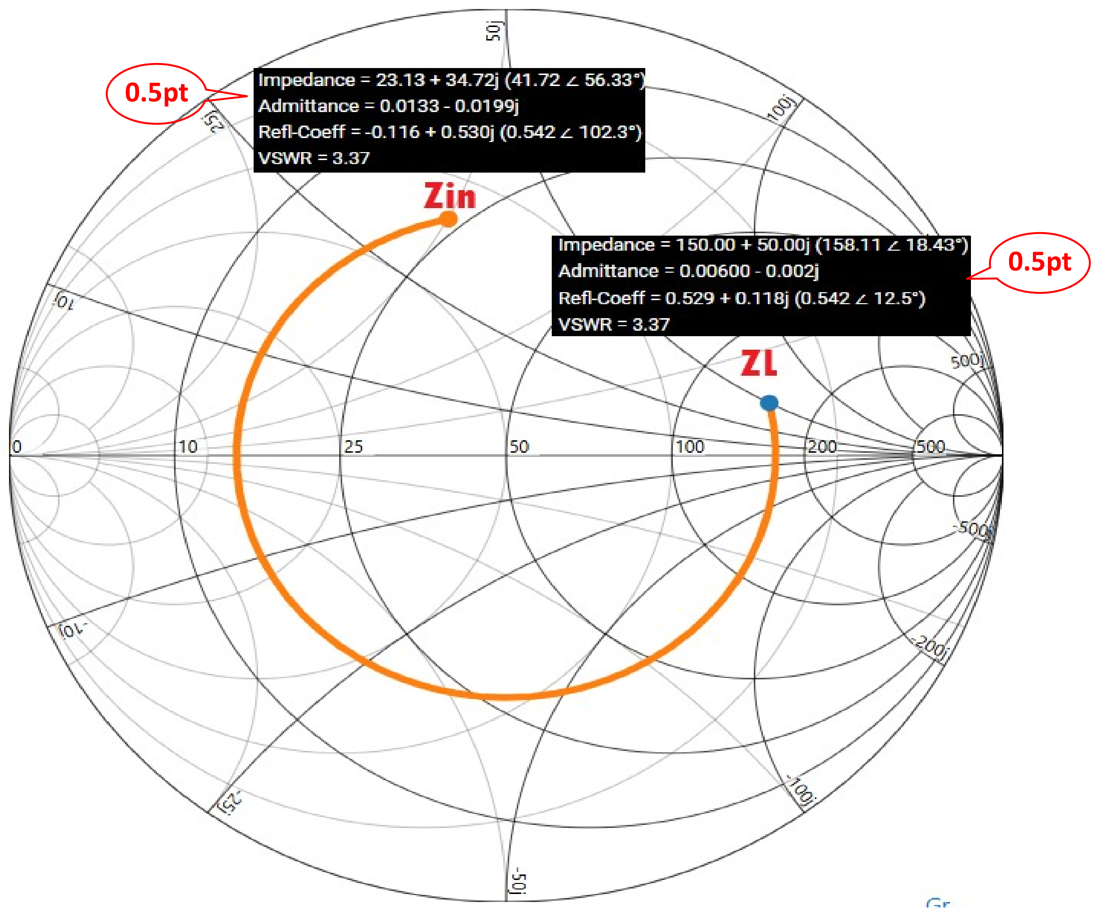
2) Standing Wave Ratio (SWR):

$$SWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + 0.54}{1 - 0.54} \approx 3.35$$ **0.5pt**

3) Input Impedance Z_{in}:

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \approx 23.08 + j34.62\Omega \approx 41.6 e^{j56.31^0}\Omega$$ **1pt**

4) Verification using the Smith chart:



- Normalization: $z_L = \dfrac{Z_L}{Z_0} = 3 + j$ **0.25pt**
- Draw the SWR circle through: $z_L = 3 + j$ **0.25pt**
- Wavelength: $\lambda = \dfrac{v}{f} = \dfrac{2\times10^8}{5\times10^6} = 40[m]$ **0.25pt**

Since $l = \dfrac{15}{40}\lambda = 0.375\lambda[m]$, the rotation on the Smith chart is

$$R = \frac{l \times 720^0}{\lambda} = \frac{0.375\times\lambda\times720^0}{\lambda} \approx 270^0$$ **0.25pt**

The normalized input impedance read from the Smith chart is: $z_{in} \approx 0.4626 + j0.6944$ **0.25pt**

$\Rightarrow Z_{in} = z_{in} \times Z_0 \approx 23.13 + j34.72[\Omega]$ **0.25pt**

Exercise 2 (7 points):

1) The radiated fields of an elementary dipole *dl*, fed by a current I_0, in the far-field expressions are:

Electric field **E**: $E_\theta = j\eta_0 \dfrac{I_0 dl}{2\lambda r}\sin(\theta)e^{-j\beta r} = j\eta_0 \dfrac{I_0 \beta dl}{4\pi r}\sin(\theta)e^{-j\beta r}$ **0.5pt**

Magnetic field **H**: $H_\varphi = \dfrac{E_\theta}{\eta_0} = j\dfrac{I_0 dl}{2\lambda r}\sin(\theta)e^{-j\beta r} = j\dfrac{I_0 \beta dl}{4\pi r}\sin(\theta)e^{-j\beta r}$ **0.5pt**

where: $\beta = 2\pi/\lambda$ is the phase constant, λ is the wavelength, *r* is the distance from the dipole to the observation point **P**, and η_0 is the intrinsic impedance of free space.

2) Radiated power density in the direction *r*:

The average radiated power density is given by the Poynting vector:

$$\vec{\wp}_{ave} = \frac{1}{2}\Re e\left\{\vec{E}\times\vec{H}^*\right\}$$

For the elementary dipole:

$$\vec{\wp}_{ave} = \frac{|E_\theta|^2}{2\eta_0}\vec{a}_r = \left(\eta_0 \frac{I_0 dl}{2\lambda r}\sin(\theta)\right)^2 \frac{1}{2\eta_0}\vec{a}_r = \left(\frac{I_0 dl}{\lambda r}\sin(\theta)\right)^2 \frac{\eta_0}{8}\vec{a}_r$$

with $\eta_0 = 120\pi\Omega$

$$\vec{\wp}_{ave} = 15\pi\left(\frac{I_0 dl}{\lambda r}\sin(\theta)\right)^2 \vec{a}_r$$ **1pt**

and $\vec{\wp}_{ave} = \wp\vec{a}_r \rightarrow \wp = 15\pi\left(\dfrac{I_0 dl}{\lambda r}\sin(\theta)\right)^2$

3) Total radiated power into all space:

The total radiated power is obtained by integrating $\vec{\wp}_{ave}$ over a sphere of radius r:

$$P_r = \iint_S \vec{\wp}_{ave} \overrightarrow{dS}\text{, with } \overrightarrow{dS} = r^2 \sin(\theta) d\theta d\varphi \vec{a}_r$$

0.5pt

Thus, $P_r = \iint_S \left(\frac{I_0 dl}{\lambda r}\sin(\theta)\right)^2 \frac{\eta_0}{8} r^2 \sin(\theta) d\theta d\varphi = \left(\frac{I_0 dl}{\lambda}\right)^2 \frac{\eta_0}{8} \int_{\theta=0}^{\pi} \sin(\theta)^3 d\theta \int_{\varphi=0}^{2\pi} d\varphi = \left(\frac{I_0 dl}{\lambda}\right)^2 \frac{\eta_0 \pi}{3}$

which gives: $P_r = \left(\frac{I_0 dl}{\lambda}\right)^2 \frac{\eta_0 \pi}{3} = 40\left(\frac{\pi I_0 dl}{\lambda}\right)^2$ 0.5pt

Radiation resistance: $P_r = \frac{1}{2} R_r I_0^2 = 40\left(\frac{\pi I_0 dl}{\lambda}\right)^2 \rightarrow R_r = 80\left(\frac{\pi dl}{\lambda}\right)^2$ 0.5pt

4) Directivity: $D(\theta,\varphi) = \frac{U(\theta,\varphi)}{P_r/4\pi} = \frac{r^2 \wp}{P_r/4\pi} = \frac{15\pi\left(\frac{I_0 dl}{\lambda}\sin(\theta)\right)^2}{10\pi\left(\frac{I_0 dl}{\lambda}\right)^2} = 1.5\sin^2(\theta)$ 0.5pt

The maximum directivity is: $D_{\max} = D\left(\theta = \frac{\pi}{2}, \varphi\right) = 1.5$ 0.5pt

5) Fields produced by dipoles **B** and **C** :

Dipole B oriented along the y-axis:

The field expressions are analogous to those of dipole A, except that the angle θ is replaced by the angle between the y-axis and the direction r.

$$\cos(\psi) = \vec{a}_\psi . \vec{a}_r = \vec{a}_x . \vec{a}_r = \sin(\theta)\sin(\varphi) \Rightarrow \sin(\psi) = \sqrt{1-\sin^2(\theta)\sin^2(\varphi)}$$

Electric field E:

$$E_B = j\eta_0 \frac{I_0 dl}{2\lambda r}\sqrt{1-\sin^2(\theta)\sin^2(\varphi)}e^{-j\beta r} = j\eta_0 \frac{I_0 \beta dl}{4\pi r}\sqrt{1-\sin^2(\theta)\sin^2(\varphi)}e^{-j\beta r}$$

0.5pt

Magnetic field H:

$$H_B = \frac{E_B}{\eta_0} = j\frac{I_0 dl}{2\lambda r}\sqrt{1-\sin^2(\theta)\sin^2(\varphi)}e^{-j\beta r} = j\frac{I_0 \beta dl}{4\pi r}\sqrt{1-\sin^2(\theta)\sin^2(\varphi)}e^{-j\beta r}$$

0.5pt

Dipole C oriented in the xy-plane:

Similarly, θ is replaced by the angle between the dipole axis and the direction r.

$$\vec{a}_\psi = \cos\left(\frac{\pi}{3}\right)\vec{a}_x + \sin\left(\frac{\pi}{3}\right)\vec{a}_y$$

$$\cos(\psi)=\vec{a}_{\psi}.\vec{a}_{r}=\cos\left(\frac{\pi}{3}\right)\sin(\theta)\cos(\varphi)+\sin\left(\frac{\pi}{3}\right)\sin(\theta)\sin(\varphi)$$

$$=\sin(\theta)\left[\cos\left(\frac{\pi}{3}\right)\cos(\varphi)+\sin\left(\frac{\pi}{3}\right)\sin(\varphi)\right]$$

$$=\sin(\theta)\cos\left(\varphi-\frac{\pi}{3}\right)$$

$$\sin(\psi)=\sqrt{1-\sin^2(\theta)\cos^2\left(\varphi-\frac{\pi}{3}\right)}$$ 0.5pt

Electric field E:

$$E_C=j\eta_0\frac{I_0 dl}{2\lambda r}\sqrt{1-\sin^2(\theta)\cos^2\left(\varphi-\frac{\pi}{3}\right)}e^{-j\beta r}=j\eta_0\frac{I_0\beta dl}{4\pi r}\sqrt{1-\sin^2(\theta)\cos^2\left(\varphi-\frac{\pi}{3}\right)}e^{-j\beta r}$$ 0.5pt

Magnetic field H:

$$H_C=\frac{E_C}{\eta_0}=j\frac{I_0 dl}{2\lambda r}\sqrt{1-\sin^2(\theta)\cos^2\left(\varphi-\frac{\pi}{3}\right)}e^{-j\beta r}=j\frac{I_0\beta dl}{4\pi r}\sqrt{1-\sin^2(\theta)\cos^2\left(\varphi-\frac{\pi}{3}\right)}e^{-j\beta r}$$ 0.5pt

Exercise 3 (3 points):

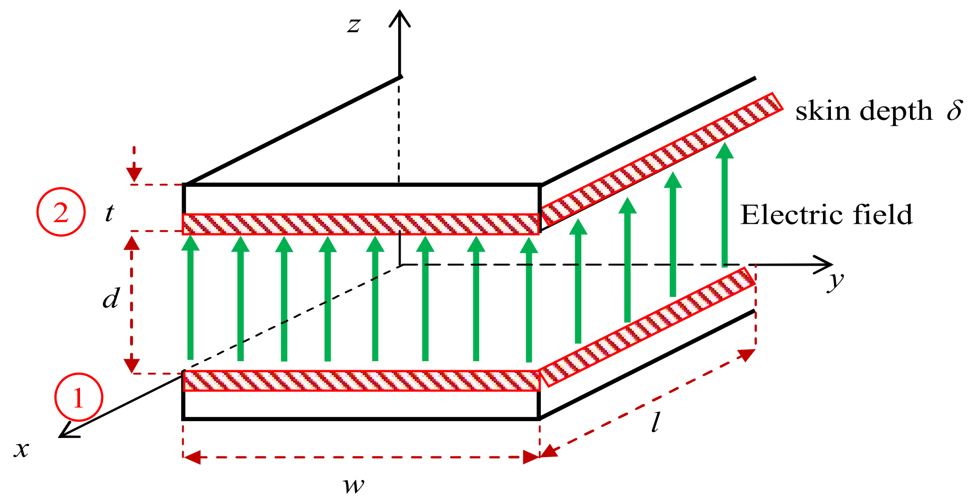
At high frequency, the current density is concentrated near the conductor surface due to the skin effect.

The effective conduction thickness is therefore equal to the skin depth δ.

Hence, the effective cross-sectional area of each conducting plate is: $S=\delta w$

where: w is the width of the plate, and δ is the skin depth.

The electric field between the plates is assumed to be uniform and directed along the z-axis: $\vec{E}=E_z\vec{a}_z$



1) Per-unit-length resistance (R_l): The total resistance of both plates of the structure is the sum of the resistances of the two conducting plates: $R = R_1 + R_2$

Since the two plates are identical: $R_1 = R_2 = \dfrac{l}{\sigma_c S}$

Substituting ($S = \delta w$): $R_1 = R_2 = \dfrac{l}{\sigma_c \delta w}$ 0.5pt

Thus, $R = \dfrac{2l}{\sigma_c \delta w}$

The resistance per-unit-length is defined as: $R_l = \dfrac{R}{l}$

Therefore, $R_l = \dfrac{2}{\sigma_c \delta w}$ 0.5pt

2) Per-unit-length capacitance (C_l)

The capacitance is defined as: $C = \dfrac{Q}{V}$ 0.25pt

Using Gauss's law: $Q = \varepsilon \iint_S \vec{E} \cdot \vec{dS}$ 0.25pt

Since: $\vec{dS} = dxdy\vec{a}_z$

and the electric field is uniform, $Q = \varepsilon E_z \int_0^l dx \int_0^w dy = \varepsilon E_z lw$

Hence, $E_z = \dfrac{Q}{\varepsilon lw}$ 0.5pt

The potential difference between the two plates is: $V = -\int_1^2 \vec{E} \cdot \vec{dl}$

Because the field is uniform across the separation distance d,: $V = E_z d$ 0.5pt

Substituting the expression of E_z : $V = \dfrac{Qd}{\varepsilon lw}$

Therefore, the capacitance becomes: $C = \dfrac{Q}{V} = \dfrac{Q\varepsilon lw}{Qd} = \dfrac{\varepsilon lw}{d}$

Finally, the per-unit-length capacitance is: $C_l = \dfrac{C}{l} = \dfrac{\varepsilon w}{d}$ 0.5pt