Defect and Diffusion Forum
ISSN: 1662-9507, Vol. 406, pp 473-480
© 2021 Trans Tech Publications Ltd, Switzerland

Submitted: 2020-04-07
Revised: 2020-04-22
Accepted: 2020-04-24
Online: 2021-01-11

Application of Associated and Non-Associated Flow Metal Plasticity for F.S.S Sheet

Oualid Chahaoui[1,a*], Houssem Soltani[2,b], Nadjoua Matougui[3,c]

[1]Engineering Sciences and Advanced Materials Laboratory (ISMA), Laghrour-Abbes University of Khenchela, P.O. Box 1252, 40004, Algeria

[2]L3M-National School of Mining and Metallurgy-Annaba, P.O. Box 233, 23000, Algeria

[a]oualid.chahaoui@gmail.com,
[b]houssem.soltani@aiesec.net, [c]nedjoua.matougui@ensmm-annaba.dz

Keywords: Yield function, anisotropic metal, Constitutive model, Non-associated flow rule, sheet metal forming

Abstract. In the last decade, several phenomenological yield criteria for anisotropic material has been proposed to improve the modeling predictions about sheet metal-forming processes. In regard to this engineering application, two proprieties of models have been used. If the yield function and the plastic potential are not same (not equal), the normality rule is non associative flow rule (NAFR), otherwise, when the stresses yield has been completely coupled to the anisotropic strain rate ratio (plastic potential), is called the associated flow rule (AFR). The non-associated flow rule is largely adopted to predict a plastic behavior for metal forming, accurately about à strong mechanical anisotropy presents in sheet metal forming processes. However, various studies described the limits of the AFR concept in dealing with highly anisotropic materials. In this study, the quadratic Hill1948 yield criteria is considered to predict mechanical behavior under AFR and NAFR approach. Experiment and modeling predictions behaviour of normalized anisotropic coefficient *r (θ)* and *σ (θ)* evolved with θ in sheet plane. and the equibiaxial yield stress $\boldsymbol{\sigma_b}$ was assumed σ_b=1 but the r_b-values was computed from Yld96 [15].

Introduction

The characterization of the drawability of rolled sheets is based on the determination of several features, which serve as a tool of mastering and understanding the behavior of those rolled sheets destined to drawing operations. For many sheet metals obtained by cold-rolling, a strong gradient of crystallographic texture exhibit in in plane and thickness of metal [1]. Especially true for ferritic steels (FSS), for two reasons: (i) continuous recrystallization occurs during hot working, which precludes randomization of grain orientations, and (ii) no phase transformation occurs upon cooling. Furthermore, such textures cannot be entirely eliminated or even modified by subsequent cold rolling and/or heat treatments [2]. The rolling sheets are characterized by the presence of three mutually orthogonal planes of orthotropic symmetry. It is notable that the rolling process promotes the existence of induced anisotropy, which greatly influences the Drawability properties and formability of sheet metals to the desired dimensions and shape [3]. For this purpose and in aim to describe the anisotropic behavior of materials, both functions (quadratic and non-quadratic) have been proposed. Initially, Hill [4], which is a simple modification of the standard von Mises plasticity to give a quadratic yield function mainly based on the assumption of an associated flow rule. It is quite interesting to see the difference in the yield locus when compared the two mode of plasticity (AFR and NAFR for yield function or plastic potential) modelled under Hill1948 criteria along different orientations in plane sheet. The mechanical parameters commonly used to characterize the anisotropic behavior of deformations and stresses in the plane sheet is the unidirectional yield stresses σ(θ) and the strain rate ratio r(θ) (the anisotropy coefficient), or Lankford coefficient, defined in unidirectional tensile tests on rectangular sheet specimens [5].

Modern Plasticity (Non-Associated Plasticity)

In the last work published by Spitzig et al. [6] and Richmond-Spitzig [7] who reported experimental data incompatible with the requirement of associated plasticity (AFR) with respect to the dependence on hydrostatic pressure (mean stress in the spherical tensor) of flow stress during a classical plasticity (associated plasticity). The final volume calculated by the components of the strain tensor based on the associated plasticity (AFR) is no significant from that measured, where plastic potential is independent from the yield function under Non-Associated Flow Rule (Non-AFR). Stoughton [8] proposed a new computational model resulting in the uncoupled predictions of stress ratios and r-values. Safaei et al [9], proposed a model based on the non-associated flow rule that adopted Barlat's yield function (Barlat et al., 2003).

Yield Function for Sheet Metals

Hill48 yield criteria. In the current study, the Hill 1948 yield criterion function, one of the most common models of metal deformation or yield plastic potential can be written in a general form as,

$$f(\sigma_{ij}) = (F+G)\,\sigma_{11}^2 + (H+F)\,\sigma_{22}^2 - 2H\sigma_{11}\sigma_{22} + 2N\sigma_{12}^2 = 2\sigma_0^2 \tag{1}$$

F, *G*, *H*, *L*, *M* and *N* anisotropy coefficients can be derived from texture components (see also [2]) or usually determined from mechanical tests (i.e.; usual uniaxial tensile as well as simple shearing), as hereafter:

where $L = M = N = 3G = 3F = 3H = \frac{3}{2}$, Hill48 yield function reduces to Mises yield function.

If σ_0^1, σ_0^2 and σ_0^3 are the yield stresses in uniaxial tension along the axes RD, TD, and ND, respectively:

$$G + H = 2\left(\sigma_0/\sigma_0^1\right)^2 \qquad F + H = 2\left(\sigma_0/\sigma_0^2\right)^2 \qquad F + G = 2\left(\sigma_0/\sigma^3\right)^2 \tag{2}$$

If σ_0^{23}, σ_0^{13} and σ_0^{12} are the simple shear stresses along the anisotropy axes:

$$L = \left(\sigma_0/\sigma_0^{23}\right)^2 \qquad M = \left(\sigma_0/\sigma_0^{13}\right)^2 \qquad N = \left(\sigma_0/\sigma_0^{12}\right)^2$$

Whenever, in plane stress state (i.e.; $\sigma_{33} = \sigma_{13} = \sigma_{23} = 0$, and $\sigma_{11}, \sigma_{22}, \sigma_{12} \neq 0$), Eq. 1 can be reduced to

$$F\sigma_{22}^2 + G\sigma_{11}^2 + H(\sigma_{11} - \sigma_{22})^2 + 2N\sigma_{12}^2 = 2\sigma_0^2 \tag{3}$$

Since formability of any sheet is characterized by $\sigma(\theta)$ mechanical parameter that is primarily related to the size and shape of grains, drawability is usually related to the $r(\theta)$*-value*, (Lankford's Parameter) which is defined as the ratio of the true strains in the width and in the thickness directions, respectively. Based on the Hill48 quadratic criterion as well as on the associated flow rule according to normality principle, relationships determining mechanical and anisotropic parameters are:

$$\sigma(\theta) = \frac{\sigma_0}{\left(F\sin^4\theta + G\cos^4\theta + H\cos^2 2\theta + 2N\sin^2\theta\cos^2\theta\right)^{1/2}} \tag{4a}$$

$$r(\theta) = \frac{H\cos^2 2\theta - (F + G - 2N)\cos^2\theta\sin^2\theta}{F\sin^2\theta + G\cos^2\theta} \tag{4b}$$

The most common method to obtain *F, G, H* and *N material* parameters of the Hill'48 model, can be calculated by two methods:

1) Associative flow rule (AFR) as the dominant theory in metal plasticity is known to **use** an identical formulation for both the yield and the plastic potential functions

i) Related to experimental yield stresses and *r*-values as follows [10].

$$F=\frac{1}{2}\left(\frac{2\sigma_0^2}{\sigma_{90}^2(1+r_{90})}\right) \qquad G=1-\frac{1}{2}\left(\frac{2\sigma_0^2 r_{90}}{\sigma_{90}^2(1+r_{90})}\right)$$
$$H=\frac{1}{2}\left(\frac{2\sigma_0^2 r_{90}}{\sigma_{90}^2(1+r_{90})}\right) \qquad N=\frac{1}{2}\left(\frac{4\sigma_0^2}{\sigma_{45}^2}-1+\frac{\sigma_0^2(r_{90}-1)}{\sigma_{90}^2(1+r_{90})}\right) \qquad (5)$$

2) Non-Associative flow rule (NAFR)

ii) Related to experimental yield stresses as follows [11].

$$F=\frac{1}{2}\left[\frac{\sigma_0^2}{\sigma_{90}^2}+\frac{\sigma_0^2}{\sigma_b^2}-1\right] \quad G=\frac{1}{2}\left[1-\frac{\sigma_0^2}{\sigma_{90}^2}+\frac{\sigma_0^2}{\sigma_b^2}\right] \quad H=\frac{1}{2}\left[1+\frac{\sigma_0^2}{\sigma_{90}^2}-\frac{\sigma_0^2}{\sigma_b^2}\right] \quad N=\frac{1}{2}\left[\frac{4\sigma_0^2}{\sigma_{45}^2}-\frac{\sigma_0^2}{\sigma_b^2}\right] \qquad (6)$$

where $\sigma_0, \sigma_{45}, \sigma_{90}$ are unidirectional yield stresses of 0°, 45° and 90° according to the rolling direction (RD). σ_b is the biaxial yield stress determined by a biaxial tensile test experiment. Noting that, all these variants imply $\sigma_{ref}=\sigma_0$

iii) Related to experimental r-values; [12].

$$F'=\frac{r_0}{r_{90}(1+r_0)} \qquad G'=\frac{1}{(1+r_0)} \qquad H'=\frac{r_0}{(1+r_0)} \qquad N'=\frac{(1+2r_{45})(r_0+r_{90})}{2r_{90}(1+r_0)} \qquad (7)$$

The Lankford parameters r_0, r_{45}, r_{90} are the anisotropic ratios of the plastic strain rate across the width of a uniaxial tension test at 0, 45, and 90 degrees to the rolling direction of the plane sheet.

Uniaxial Tensile Test

Uniaxial tensile tests were conducted to determine two essential parameters, the yield stress ***σ (θ)*** and the ***r (θ)*** of the overall sheet. The sheet samples prepared along three orientations 0°=RD, 45°=DD and 90°=TD from the rolling direction (RD) were investigated. The anisotropic coefficients of this rolled sheet were also calculated for the 18% pre-strain level and the results were tabulated in Table1.

Table 1 Material Mechanical Property for FS Steel in three directions

Direction	v	σ_e *(0.2% offset) [MPa]*	r
0°	0.3	278	0.7
45°		283	1..41
90°		271	0.82

Fig. 1 shows the uniaxial hardening curves for specimens extracted at three different orientations (RD, DD and TD). The yield curves in TD and RD are even crossing each other. The experimental data were taken from Chahaoui et al. (2013) [13,14].

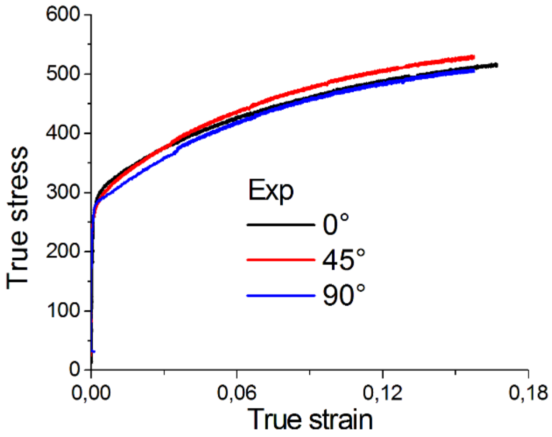

Fig. 1 Hardening curves for FS Steel

The normalized flow stresses (yield stress) and r-value for different directions are presented in Table 2. Yield stresses for each direction were then normalized by the mono-directional yield stress along the rolling direction.

Table 2 The normalized tensile yield stress by the rolling direction uniaxial yield stress

Yield stress	σ_0/σ_u	σ_{45}/σ_u	σ_{90}/σ_u	σ_b/σ_u
	1	1.021	0.97	1
r-value	r_0/r_0	r_{45}/r_0	r_{90}/r_0	r_b
	1	2.01	1.17	0.8612

Note that in this work, the equibiaxial yield stress $\boldsymbol{\sigma_b}$ was assumed $\sigma_b = 1$ and the r_b-value was computed from Yld96.

Parametric Study

In the first step, it's interesting to begin very simply by evaluate one variable parametrically and analyze the results of each parameter variation. To quantify the direct influence of the *r-value* on the contour dependency of the yield stress of Hill48 yield criteria. The uniaxial yield values (0° and 90°) for each *Lankford coefficients* (r_o and r_{90}) are carried out (fig. 2), with keeping the value of r_{45} constant and equal to the unity.

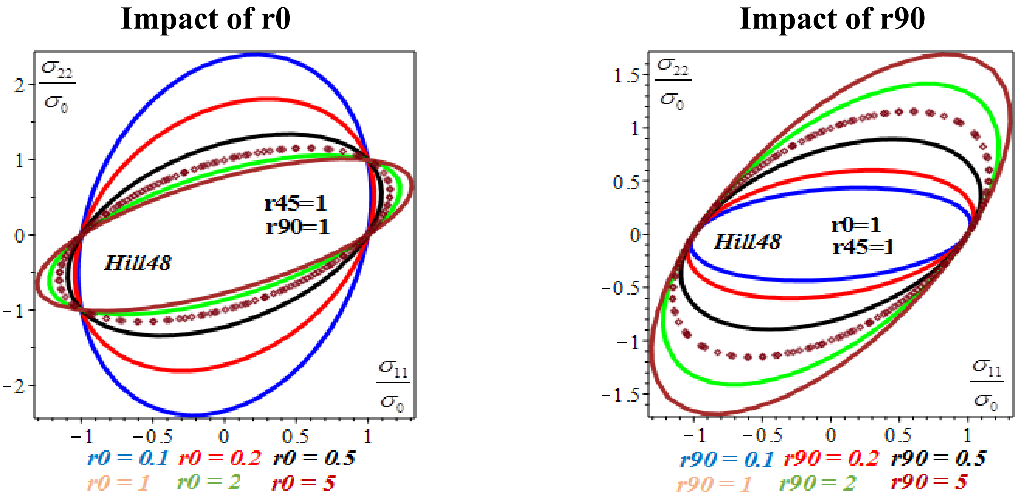

Fig. 2 Impact of *r0* and *r90* on the load surface of Hill48, $\sigma_{ref} = \sigma_0$ is fixed and normalized to 1 MPa in the RD (0°)

Note that, it is well established that variation in *ro* and *r90* affects the way large the normalized transverse flow stress σ_{22}/σ_0 (Lankford coefficient is directly proportional to stress) for Hill48. A small variation in *ro* and *r90* values gives a strongly influences the behavior of the material

Determination of Anisotropy Coefficients for Different Yield Functions

The resulting anisotropic coefficients of Hill's 1948 (*F, G, H* and *N*) for as-received material.

1- Calculated anisotropy parameters identified for AFR

Associted (σ-*r-values*)	F	G	H	N
	0,5791	0,5250	0,4749	1,5039

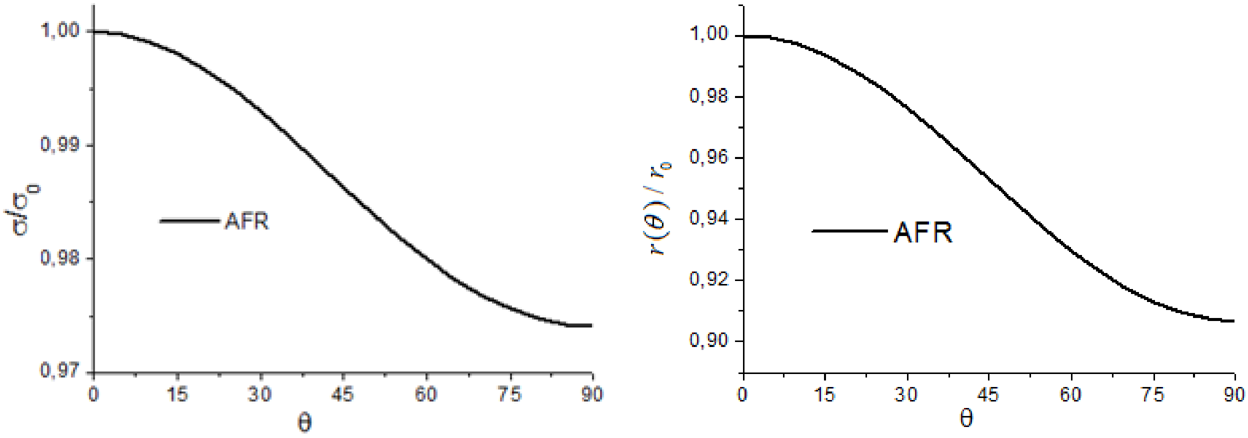

Fig. 3 The normalized strain rate ratio $r(\theta)$ and yield stress $\sigma(\theta)$ in the sheet plane showing the angular dependence for AFR

2- Calculated anisotropy parameters identified for NAFR

3- Calculated anisotropy parameters identified for yield function

Non-Associted (σ-Function)	F	G	H	N
	0,5270	0,4729	0,5270	1,4336

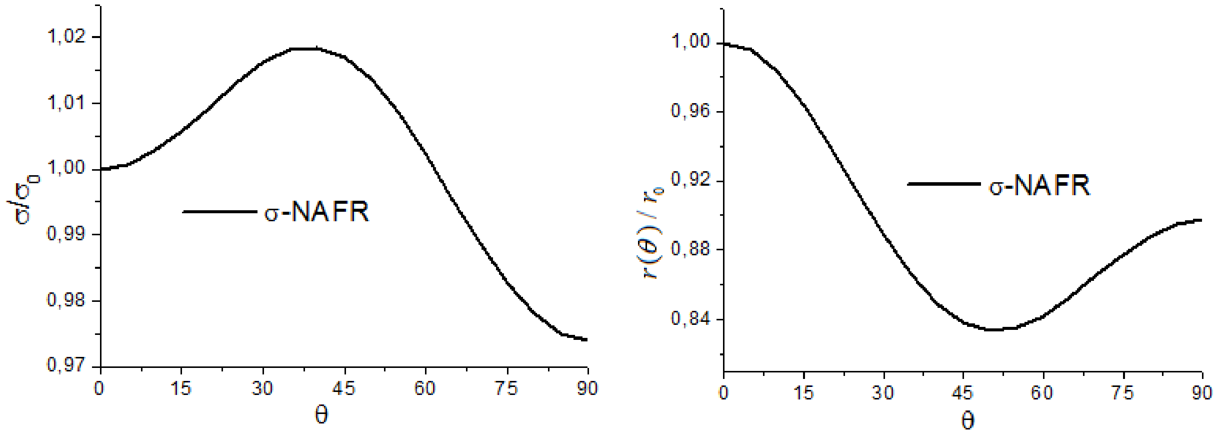

Fig. 4 The normalized strain rate ratio $R(\theta)$ and yield stress $\sigma(\theta)$ in the sheet plane showing the angular dependence for NAFR (σ-Function)

4- Calculated anisotropy parameters identified for plastic potential

Non-Associted (r-values)	F	G	H	N
	0,5021	0,5882	0,4117	2,0826

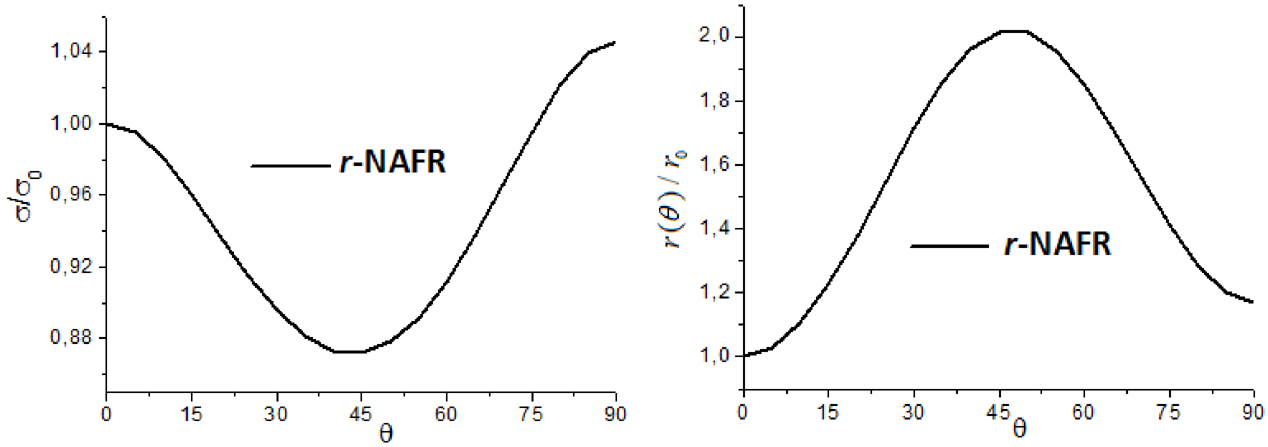

Fig. 5 The normalized strain rate ratio $r(\theta)$ and yield stress $\sigma(\theta)$ in the sheet plane showing the angular dependence for NAFR (r-values)

As a partial result of this comparison, the non-associated flow rule presented a better adjustment of the results with the experimental (see Figure 6 below).

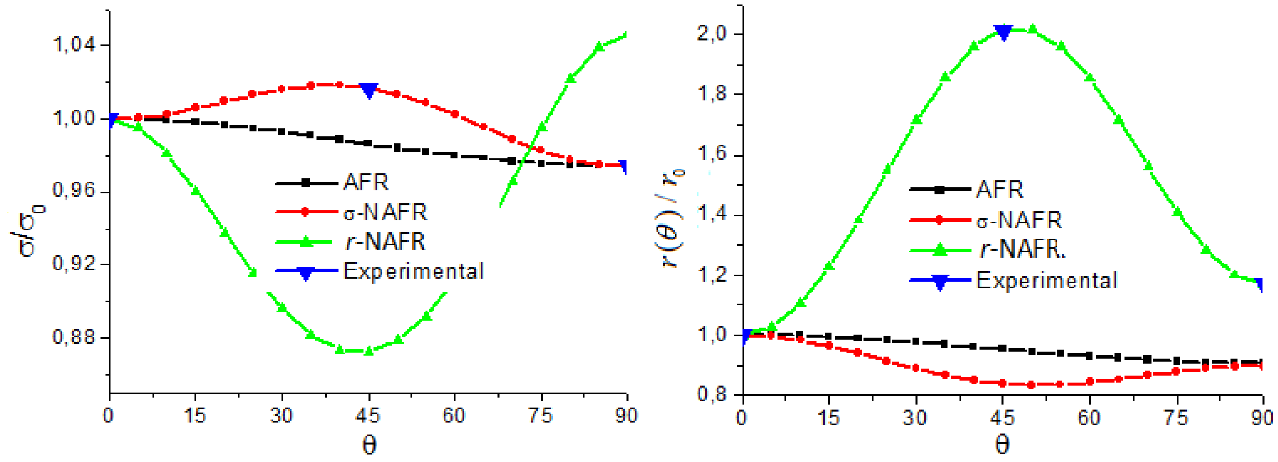

Fig. 6 The normalized strain rate ratio $r(\theta)$ and yield stress $\sigma(\theta)$ in the sheet plane showing the angular dependence for AFR and NAFR

Conclusions

In this paper, the assessment of the two applications of plasticity (associated and non-associated flow rules) for Hill48 yield criteria has been presented.

Initially, the analysis of the classical case of a homogeneous sheet has shown that the results can be significantly different depending on whether the test frame of plasticity applied to this type of sheets, associated plasticity (commonly accepted hypothesis = no influence of hydrostatic pressure) or not associated (change of volume during deformation).

For this characterized sheet metal, the evolution of the anisotropy parameters of the two tensors (stress-strain) in the plane of the sheet metal, shows the importance of identifying these quantities (F, G, H and N) in associated and non-associated plasticity.

- For the associated plasticity, the variations of the curves diverge from the experimental one.
- For non-associated plasticity, consistent variations and better agreement with experimental results, the decoupling of the flow stress tensor from the strain tensor gives a good prediction. For instance, the NAFR *r(θ)*-based Hill48 model can result in an accurate prediction of Lankford coefficients at 0°, 45° and 90° and the NAFR *σ(θ)*-based Hill48 model results in an exact fit to the initial yield.

Parameters of Hill'48 yield criteria, were calibrated based on tensile flow stresses and r-values in tensile tests in three directions (0°,45° and 90°) and the equibiaxial yield stress $\boldsymbol{\sigma_b}$ was assumed $\sigma_b = 1$ but the r_b-value was computed from Yld96 [15] .

References

[1] R.P.R. Cardoso, O.B. Adetoro, A generalisation of the Hill's quadratic yield function for planar plastic anisotropy to consider loading direction, International Journal of Mechanical Sciences 128–129 (2017) 253–268

[2] O. Chahaoui, M. L. Fares, D. Piot, F. Montheillet, Monoclinic effects and orthotropic estimation for the behaviour of rolled sheet, Journal of Materials Science 46: (2011)1655-1667

[3] N. Brinis, B. Regaiguia, O. Chahaoui, N. Maatougui, M. L Fares, Modelling Mechanical Properties of AISI 439-430Ti Ferritic Stainless-Steel Sheet, Journal of Solid Mechanics Vol. 11, No. 2, (2019). pp 425-439. DOI: 10.22034/JSM.2019.665911

[4] R. Hill, A theory of yielding and plastic flow of anisotropic materials. Proceedings: Mathematical, Physical and Engineering Science, Royal Society London, A193 (1948) 281-297

[5] Monica Iordache1, Mihaela Teaca1, 2, Isabelle Charpentier2, Marion Martiny2, Gérard Ferron2 'Identification of sheet metal plastic anisotropy, and optimization of initial blank shape in deep drawing' the annals of "dunărea de jos" university of galaţi fascicle v, technologies in machine building, ISSN (2009) 1221- 4566

[6] W.A Spitzig, R.J Sober, and O Richmond, Pressure dependence of yielding and associated volume expansion in tempered martensite, Acta Metallurgica, 23(7): (1975) 885 – 893

[7] W.A. Spitzig and O. Richmond, The effect of pressure on the flow stress of metals, Acta Metallurgica, 32(3):457 – 463, 1984

[8] T.B. Stoughton, A non-associated flow rule for sheet metal forming, International Journal of Plasticity, 18(5–6): (2002) 687–714

[9] M. Safaei, M. G., Lee, S. L. Zang, and W. D. Waele, An evolutionary anisotropic model for sheet metals based on non-associated flow approach, Comput. Mater. Sci. (2014) 81, 15-29

[10] H. Aretz, Numerical analysis of diffuse and localized necking in orthotropic sheet metals, International Journal of Plasticity 23 (2007) 798–840

[11] O. Cazacu, F. Barlat, Generalization of Drucker's yield criterion to orthotropy, Mathematics and Mechanics of Solids 6 (2001) 613–630

[12] M. Kawka, A. Makinouchi, Plastic anisotropy in FEM analysis using degenerated solid element, Journal of Materials Processing Technology 60 (1996) 239–242

[13] O. Chahaoui, M.L. fares, D. Piot, F. Montheillet, Mechanical Modeling of the Macroscopic Behavior for Anisotropic and Heterogeneous Alloys, Metals and Materials International. Vol. 19, No. 5 (2013), pp. 1005~1019. doi: 10.1007/s12540-013-5013-3

[14] N. Brinis, B. Regaiguia, O. Chahaoui, N. Maatougui, M. L Fares, Modelling Mechanical Properties of AISI 439-430Ti Ferritic Stainless-Steel Sheet, Journal of Solid Mechanics Vol. 11, No. 2, (2019) pp 425-439. DOI: 10.22034/JSM.2019.665911

[15] F Barlat, R.C Becker, T. Hayashida,Y. Maeda, M. Yanagawa, K. Chung, J.C Brem, D.J Lege, K Matsui., S.J.Murtha, S Hattori., Yielding description of solution strengthened aluminium alloys, International Journal of Plasticity 13: ,(1997) 185–401