| University Abbes LAGHROUR Khenchela. | | | |
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| Faculty of sciences & Technology | | Mechanical Department : Materials Engineering | |
| Level: 3 year-MC | Final Exam : Asservissement& Regulation | Time : 1H30. | Date : 01-20-20 |

**Exercice1(09pts):**

Let a system be governed by the following differential equation, assuming null initial conditions,

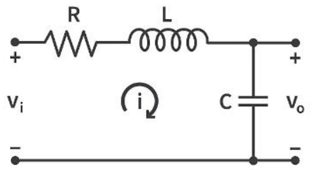
$$\ddot{y}(t) + 2\dot{y}(t) - 15y(t) = \dot{x}(t) + x(t)$$

1. Find the transfer function: $H(s) = \frac{Y(s)}{X(s)}$
2. Find the zeros and poles
3. Demonstrate that $H(s)$ can be put as $H(s)=\frac{a}{s-3} + \frac{b}{s+5}$, find $a$ and $b$
4. Given that $x(t)$ is a Dirac delta function, compute the output $y(t)$ of the system.

**Exercise 2(4pts):**

1. Find the transfer function for the figure bellow

$$H(s) = \frac{V_O(s)}{V_i(s)}$$

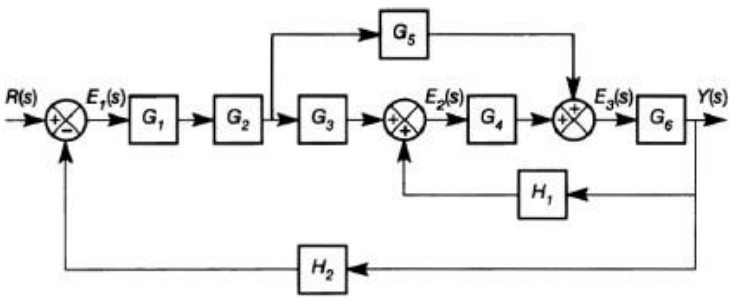



2. Prove that H(s) can be put in the form

3. $H(s) = \frac{k}{s^2+2\zeta\omega_0 s+{\omega_0}^2}$

4. Find k, $\zeta$, and $\omega_0$ as functions of $R, L,$ and $C$.

**Exercice3(7pts):** Simplify the block diagram below into a single block relating $Y(s)$ to $R(s)$.



# *Solution*

**Exercice1:**

$$\ddot{y}(t) + 2\dot{y}(t) - 15y(t) = \dot{x}(t) + x(t)$$

*Apply the Laplace transform for left and Right sides of differential equation, we get*

$\mathcal{L}[\ddot{y}(t) + 2\dot{y}(t) - 15y(t)] = \mathcal{L}[\dot{x}(t) + x(t)]$, since all initial conditions are zeros, we obtain

1pt

$$s^2Y(s) + 2sY(s) + 15Y(s) = sX(s) + X(s),$$

$$(s^2 + 2sY + 15)Y(s) = (s + 1)X(s),$$ 1pt

The transfer function for above system is

$$\boldsymbol{H(s)} = \frac{\boldsymbol{Y(s)}}{\boldsymbol{X(s)}} = \frac{\boldsymbol{s+1}}{\boldsymbol{s+2s-15}} = \frac{\boldsymbol{N(s)}}{\boldsymbol{D(s)}}$$ 1pt

- Zeros: are solution of $N(s) = s + 1 = 0,\ z_1 = -1$ 1pt
- Poles are solutions of D(s)= $+2s - 15 = 0$, *we facotorize D(s) we get*

$$D(s) = (s - p_1)(s - p_2) = 0,$$

$$D(s) = (s - 3)(s + 5) = 0$$

*Solutions*, $\boldsymbol{p}_1 = 3$, 0.5pt $\boldsymbol{p}_2 = -5$ 0.5pt

$$\frac{\boldsymbol{s+1}}{\boldsymbol{s+2s-15}} = \frac{\boldsymbol{s+1}}{(\boldsymbol{s-3})(\boldsymbol{s+5})} = \frac{\boldsymbol{A}}{\boldsymbol{s-3}} + \frac{\boldsymbol{B}}{\boldsymbol{s+5}}$$

**.** Consequently,

1pt 1pt

$$\boldsymbol{H(s)} = \frac{1}{2}\left(\frac{1}{\boldsymbol{s}-3} + \frac{1}{\boldsymbol{s}+5}\right)$$

If the input x(t) is Dirac impulse, then $X(s) = 1$,

$$Y(s) = H(s)X(s) = \frac{1}{2}\left(\frac{1}{s-3} + \frac{1}{s+5}\right)$$

Applying inverse Laplace transform, to obtain y(t)= ?

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{2}\left(\frac{1}{s-3}+\frac{1}{s+5}\right)\right\} = \frac{1}{2}\mathcal{L}^{-1}\left(\frac{1}{s-3}\right)+\frac{1}{2}\mathcal{L}^{-1}\left(\frac{1}{s+5}\right)$$ 1pts

$$y(t) = \frac{1}{2}\left[e^{-3t}+\frac{1}{2}e^{5t}\right]u(t)$$ 1pt

**Exercice2 :**

According to voltage divider, we write the complex form

$V_O(s) = \frac{Z_c}{R+Z_L+Z_c}V_i(s)$, where $z_L = jL\omega = Ls$, and $z_C = \frac{1}{jC\omega} = \frac{1}{Cs}$. Then

2pts

$$H(s) = \frac{V_O(s)}{V_i(s)} = \frac{Z_c}{R+Z_L+Z_c} = \frac{\frac{1}{Cs}}{R+Ls+\frac{1}{Cs}} = \frac{1}{LCs^2+RCs+1} = \frac{1/LC}{S^2+(R/L)s+(1/LC)} =$$

$$\frac{1/LC}{s^2+2\zeta\omega_n s+\omega_n^2} \Rightarrow$$

Exercice3 :