

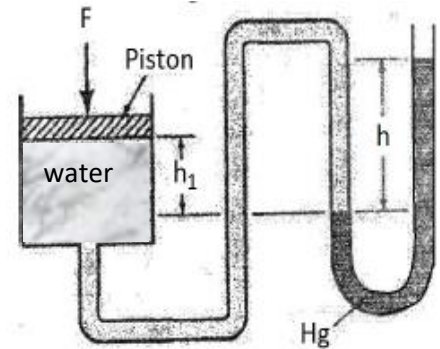
Exam: Fluid mechanics

Exercise 1

A piston of cross-sectional area $S = 0.07 \text{ m}^2$ is mounted on the top of a cylindrical tank filled with water.

A U-tube mercury manometer is connected to the bottom of the tank, while the other limb is open to the atmosphere.

For $h_1 = 60 \text{ mm}$ and $h = 100 \text{ mm}$, determine the magnitude of the force exerted by the piston, assuming that the weight of the piston is negligible.



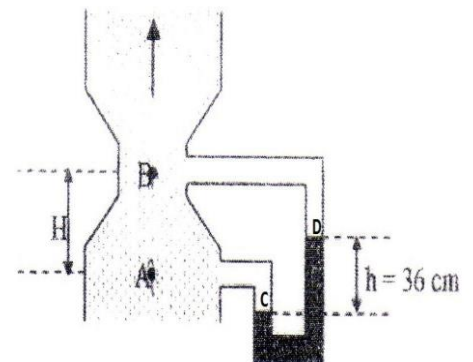
Exercise 2

Water flows upward through a Venturi tube as shown in the figure. The following data are given:

- The diameter of the tube at section A is $D_A = 30 \text{ cm}$.
- The diameter of the tube at section B is $D_B = 15 \text{ cm}$.
- The difference in mercury levels in the differential manometer is $h = 36 \text{ cm}$.

Questions:

1. Calculate the velocity of water at section B.
2. Determine the volumetric flow rate of water through the tube.



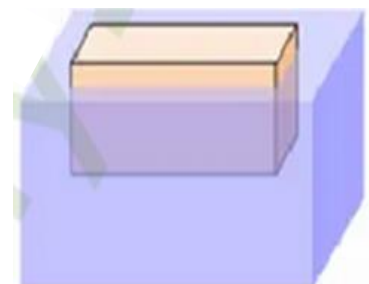
Exercise 3

A solid block floats at the surface of water and has the shape of a rectangular parallelepiped.

Its dimensions are: height $h = 20 \text{ cm}$, length $L = 60 \text{ cm}$, and width $l = 20 \text{ cm}$.

Given that the immersed portion of the block has a height $h' = 3 \text{ cm}$:

- **Determine the mass and the density of the block.**



The following data are given:

Acceleration due to gravity: $g = 10 \text{ m/s}^2$.

Densities: $\rho_{\text{water}} = 1 \text{ g.cm}^{-3}$, $\rho_{\text{Hg}} = 13.6 \text{ g.cm}^{-3}$

Exam correction

Exercise 1 (6 points)

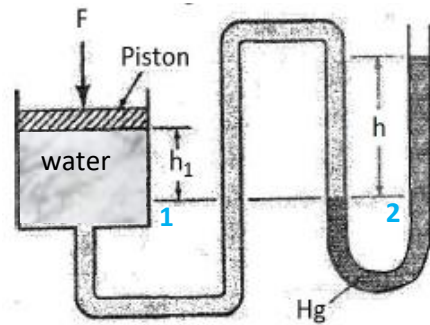
$$P_1 = P_{atm} + \frac{F}{S} + \rho_w g h_1 \quad \boxed{1.5}$$

$$P_1 = P_2 \quad \boxed{1.5}$$

$$P_2 = P_{atm} + \rho_{Hg} g h \quad \boxed{1.5}$$

$$\Rightarrow F = (\rho_{Hg} g h - \rho_w g h_1) S$$

$$F = 910 \text{ N} \quad \boxed{1.5}$$



Exercise 2 (8 points)

From Bernoulli equation,

$$P_A - P_B = \frac{1}{2} \rho_w (v_B^2 - v_A^2) + \rho_w g (Z_B - Z_A) \quad \boxed{1}$$

$$= \frac{1}{2} \rho_w (v_B^2 - v_A^2) + \rho_w g H \dots\dots\dots(1)$$

$$P_C = P_A + \rho_w g (Z_A - Z_C) \quad \boxed{0.5}$$

$$P_D = P_B + \rho_w g (Z_B - Z_D) \quad \boxed{0.5}$$

$$P_C - P_D = P_A - P_B + \rho_w g (Z_A - Z_C - Z_B + Z_D)$$

$$= P_A - P_B + \rho_w g (h - H) \quad \boxed{1}$$

On the other hand :

$$P_C - P_D = \rho_{Hg} g h \quad \boxed{1}$$

Then

$$P_A - P_B = \rho_{Hg} g h - \rho_w g (h - H) \dots\dots\dots(2)$$

By comparing equations (1) and (2),

$$\frac{1}{2} \rho_w (v_B^2 - v_A^2) + \rho_w g H = \rho_{Hg} g h - \rho_w g (h - H)$$

$$v_B^2 - v_A^2 = \frac{2gh(\rho_{Hg} - \rho_w)}{\rho_w} \dots\dots\dots(3) \quad \boxed{1}$$

And from continuity equation,

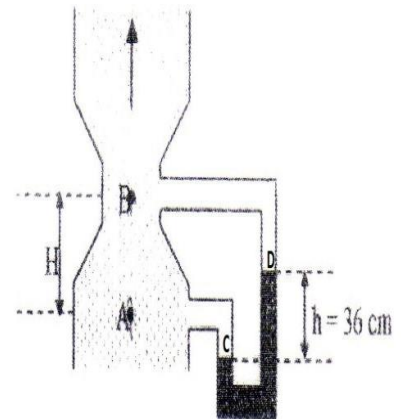
$$Q_V = S_A v_A = S_B v_B \Rightarrow v_A = \frac{S_B v_B}{S_A} \quad \boxed{1}$$

By substituting into equation (3),

$$v_B^2 = \frac{2gh(\rho_{Hg} - \rho_w)}{\rho_w \left(1 - \frac{S_B^2}{S_A^2}\right)}$$

$$v_B = 9.84 \text{ m/s} \quad \boxed{1}$$

$$Q_V = S_B v_B = 0.174 \text{ m}^3/\text{s} \quad \boxed{1}$$



Exercise 3 (6 points)

From Archimedes' principle,

The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body,

$$F_B = m_w g = \rho_w \cdot L \cdot l \cdot (h - h') \cdot g = 1000 \times 0.6 \times 0.2 \times (0.2 - 0.03) \times 10 = 204 \text{ N} \quad 1.5$$

The block is in equilibrium under the action of two forces: Archimedes' buoyant force and the weight of the block

$$1.5 \quad F_B = m_{block} \cdot g \quad \Rightarrow \quad m_{block} = \frac{F_B}{g} = 20.4 \text{ kg} \quad 1$$

$$m_{block} = \rho_{block} \cdot V_{block} \quad \Rightarrow \quad \rho_{block} = \frac{m_{block}}{L \cdot l \cdot h} = \frac{20.4}{0.6 \times 0.2 \times 0.2} \quad 1$$

$$\rho_{block} = 850 \text{ kg/m}^3 \quad 1$$