Abbes Laghrour University of Khenchela

Department of Mathematics

L2 Mathematics

Subject: *probability.* 20/05/2025

EXAM

Exercise 1.

The probability distribution of the random variable X is given by the following table

X values	0	2	3
Probabilities	0.3	0.1	0.6

Let $Y = 3(X-1)^2$.

(a) Calculate the expectation of X?

(b) Calculate the variance of X?

(c) Calculate the expectation of Y?

(d) Let F be the distribution function of Y. Calculate $F_Y(7)$?

Exercise 2.

Suppose that $X \sim$ Binomial(n, 0.5). Find the probability mass function of $Y = 2X$.

Exercise 3.

Let X be a continuous r.v. whose probability density f is defined by:

$$f(x) = \begin{cases} \frac{6}{5}x^2 & if \quad x \in [0,1] \\ \frac{6}{5}(2-x) & if \quad x \in]1,2] \\ 0 & otherwise. \end{cases}$$

1) Determine $P(X < 0.5), P(0.5 \leq X < 1.5), P\left(|X-1| < \frac{1}{4}\right)$.
2) Determine the distribution function F of X.
3) Find the distribution function of the random variable Y such that $Y = 2X - 1$.

Exercise 4 :

You roll two fair dice. If the sum of the dice is greater than 9, you win 100$. If the sum is 9 or less you get to roll again. On the second roll, if your sum of dice is greater than 9, you win 50$, otherwise you win nothing. Let X be the random variable of how much money you win by playing this game.

(a) Construct a probability model for X, i.e. make a probability table.

(b) What is the expected amount you will win playing the game?

تمرين 4

نرمي حجري نرد مجاتسين. إذا كان مجموعهما أكبر من 9، تربح 100 دولار. إذا كان المجموع 9 أو أقل، يمكنك رمي النرد مرة أخرى. في الرمية الثانية، إذا كان مجموع حجري النرد أكبر من 9، تربح 50 دولارًا، وإلا فلن تربح شيئًا

ليكن X هو المتغير العشوائي لمقدار المال الذي تربحه من لعب هذه اللعبة .

أ) أنشئ جدول احتمالات X

ب) ما هو المبلغ المتوقع ربحه من لعب هذه اللعبة؟

Solution

Exercise 1 : **(4pts)**

(a) **Solution:** We first make the probability tables

X	0	2	3
prob.	0.3	0.1	0.6
Y	3	3	12

So, $E[X] = 0 \cdot 0.3 + 2 \cdot 0.1 + 3 \cdot 0.6 = 2$

(b) **Solution:** $E[X^2] = 0 \cdot 0.3 + 4 \cdot 0.1 + 9 \cdot 0.6 = 5.8 \Rightarrow \text{Var}(X) = E[X^2] - E[X]^2 = 5.8 - 4 = 1.8.$

(c) **Solution:** $E[Y] = 3 \cdot 0.3 + 3 \cdot 0.1 + 12 \cdot 6 = 8.4.$

(d) **Solution:** From the table we see that $F_Y(7) = P(Y \leq 7) = 0.4.$

Exercise 2: **(2pts)**

Solution: For $y = 0, 2, 4, \ldots, 2n,$

$$P(Y = y) = P(X = \frac{y}{2}) = \binom{n}{y/2} \left(\frac{1}{2}\right)^n.$$

Exercise 3: **(8pts)**

1) $P(X < 0.5) = \frac{6}{5}\int_0^{0.5} t^2 dt = \frac{1}{20}.$

$P(0.5 \leq X < 1.5) = \frac{6}{5}\int_{0.5}^{1} t^2 dt + \frac{6}{5}\int_1^{1.5} (2-t)\, dt = \frac{4}{5}.$

$P\left(|X-1| < \frac{1}{4}\right) = \frac{6}{5}\int_{\frac{3}{4}}^{1} t^2 dt + \frac{6}{5}\int_1^{\frac{5}{4}} (2-t)\, dt = \frac{79}{160}.$

2) $F_X(x) = \begin{cases} 0 & for \quad x < 0; \\ \frac{2}{5}x^3 & for \quad 0 \leq x \leq 1; \\ \frac{6}{5}\left(2t - \frac{t^2}{2}\right) - \frac{7}{5} & for \quad 1 < x \leq 2; \\ 1 & for \quad x > 2. \end{cases}$

3) $F_Y(y) = P(Y \leq y) = P\left(X \leq \frac{y+1}{2}\right)$

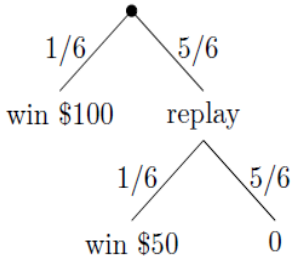
$$F_Y(y) = \begin{cases} 0 & for \quad y < -1; \\ \frac{2}{5}\left(\frac{y+1}{2}\right)^3 & for \quad -1 \leq y \leq 1; \\ \frac{6}{5}\left(2\left(\frac{y+1}{2}\right) - \frac{1}{2}\left(\frac{y+1}{2}\right)^2\right) - \frac{7}{5} & for \quad 1 < y \leq 3; \\ 1 & for \quad y > 3. \end{cases}$$

Exercise 4: (6pts)

(a)

sum of dice	2	3	4	5	6	7	8	9	10	11	12
probability	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

So the probability of rolling less than or equal to 9 is $30/36 = 5/6$, and the probability of 10 or more is $6/36 = 1/6$. The probabilities for various outcomes of the game can be computed from the following tree.



The table of probabilities and outcomes is

outcome	$100	$50	0
prob.	1/6	5/36	25/36

(b)

Solution: This can be read from the last table in **(a)** : it's

$$\text{Expected payoff} = (25/36)\cdot\$0 + (5/36)\cdot\$50 + (1/6)\cdot\$100 = \boxed{\$850/36 = \$23\frac{11}{18} \approx \$23.61}$$