

Solutions to the Final Exam in Fundamentals of Electrotechnics 02

Exercise 01 (Transformer)

Given Data

- No-load test: $V_{10} = V_{1n} = 220 \text{ V}$, $I_{10} = 0.51 \text{ A}$, $V_{20} = 82.4 \text{ V}$, $P_{10} = 17.5 \text{ W}$, $f = 50 \text{ Hz}$
- Short-circuit test: $V_{1cc} = 50 \text{ V}$, $I_{2cc} = I_{2n} = 10 \text{ A}$, $P_{1cc} = 63 \text{ W}$

1. No-load Test Wiring Diagram (Description)

During the no-load test, the primary is energized at rated voltage while the secondary is left open. The instruments required are:

- Voltmeter on primary: measures V_{10}
- Ammeter in series with primary: measures I_{10}
- Wattmeter with voltage input across primary: measures P_{10}
- Voltmeter across open secondary: measures V_{20}

2. Transformation Ratio m

$$m = \frac{V_{20}}{V_{10}} = \frac{82.4}{220} \approx \boxed{0.3745}$$

3. Purpose of Reduced Voltage in Short-circuit Test

The short-circuit test is performed at reduced voltage to avoid magnetic saturation in the core. Since the magnetic path is shorted by the windings, the nominal current can be reached at a lower voltage, preventing core losses and ensuring safety.

4. Meaning of P_{1cc}

$$\boxed{P_{1cc}} = \text{Total copper losses (in windings)}$$

During this test, core losses are negligible.

5. Secondary Winding Resistance r_s

$$r_s = \frac{P_{1cc}}{I_{2n}^2} = \frac{63}{10^2} = \boxed{0.63 \Omega}$$

6. Secondary Impedance z_s and Reactance x_s

$$z_s = m \cdot \frac{V_{1cc}}{I_{2n}} = 0.3745 \cdot \frac{50}{10} = \boxed{1.8725 \Omega}$$
$$x_s = \sqrt{z_s^2 - r_s^2} = \sqrt{1.8725^2 - 0.63^2} \approx \boxed{1.763 \Omega}$$

7. Load Voltage V_2 under Resistive Load

Assuming unity power factor:

$$\Delta V = r_s \cdot I_{2n} = 0.63 \cdot 10 = 6.3 \text{ V}$$
$$V_2 = V_{20} - \Delta V = 82.4 - 6.3 = \boxed{76.1 \text{ V}}$$

8. Transformer Efficiency and Maximum Efficiency

Output Power:

$$P_u = V_2 \cdot I_{2n} = 76.1 \cdot 10 = 761 \text{ W}$$

Total Losses:

$$P_{loss} = P_{core} + P_{cu} = 17.5 + 63 = 80.5 \text{ W}$$

Input Power:

$$P_{in} = P_u + P_{loss} = 761 + 80.5 = 841.5 \text{ W}$$

Efficiency:

$$\eta = \frac{P_u}{P_{in}} = \frac{761}{841.5} \approx \boxed{90.4 \%}$$

Maximum Efficiency Condition:

$$P_{cu} = P_{core} \Rightarrow I^2 r_s = 17.5 \Rightarrow I_{opt} = \sqrt{\frac{17.5}{0.63}} \approx 5.26 \text{ A}$$

$$P_u^{max} = V_2 \cdot I_{opt} \approx 76.1 \cdot 5.26 \approx 400 \text{ W}$$

$$P_{in}^{max} = 400 + 17.5 + 17.5 = 435 \text{ W}$$

$$\eta_{max} = \frac{400}{435} \approx \boxed{91.95 \%}$$

Exercise 02 (MAS)

Given Data

- Motor: 220/380 V, 4 poles, 50 Hz
- Supply: 127/220 V, 50 Hz (line-to-neutral / line-to-line)
- No-load test (at synchronous speed):
 - $P_1 = 1160 \text{ W}$, $P_2 = -660 \text{ W}$
 - $I_0 = 6.8 \text{ A}$
- Load test:
 - $I = 22.2 \text{ A}$
 - Slip $g = 6\% = 0.06$
 - Absorbed power $P_{abs} = 6700 \text{ W}$
- Stator phase resistance: $R_s = 1 \Omega$

1. Stator Coupling

The motor is rated at 220/380 V and supplied by a 127/220 V network. Since the line voltage matches the lower value (220 V), the stator is connected in **delta** (Δ).

2. No-load Operation Calculations

a) Synchronous Speed

$$N_s = \frac{60 \cdot f}{p} = \frac{60 \cdot 50}{2} = \boxed{1500 \text{ rpm}} \quad (4 \text{ poles})$$

b) Total No-load Power

$$P_0 = P_1 + P_2 = 1160 - 660 = \boxed{500 \text{ W}}$$

Stator copper losses during no-load:

$$P_{cu0} = 3 \cdot R_s \cdot \left(\frac{I_0}{\sqrt{3}} \right)^2 = 3 \cdot 1 \cdot (3.925)^2 \approx 3 \cdot 15.4 = 46.2 \text{ W}$$

Total mechanical and iron losses:

$$P_{fe} + P_{mec} = P_0 - P_{cu0} = 500 - 46.2 = 453.8 \text{ W}$$

Assuming they are equal:

$$P_{fe} = P_{mec} = \frac{453.8}{2} = \boxed{226.9 \text{ W}}$$

3. On-load Operation Calculations

a) Rotor Speed

$$N = N_s(1 - g) = 1500 \cdot (1 - 0.06) = \boxed{1410 \text{ rpm}}$$

b) Output Power and Useful Torque

Stator copper losses:

$$P_{cu} = 3 \cdot R_s \cdot \left(\frac{I}{\sqrt{3}} \right)^2 = 3 \cdot 1 \cdot (12.81)^2 = 3 \cdot 164.13 = 492.39 \text{ W}$$

Input power:

$$P_{in} = P_{abs} = 6700 \text{ W}$$

Iron and mechanical losses from no-load:

$$P_{fe} = P_{mec} = 226.9 \text{ W}$$

Total transmitted power to the rotor:

$$P_{tr} = P_{in} - P_{cu} - P_{fe} - P_{mec} = 6700 - 492.39 - 226.9 = 5980.71 \text{ W}$$

Rotor copper losses:

$$P_{jr} = g \cdot P_{tr} = 0.06 \cdot 5980.71 = 358.84 \text{ W}$$

Useful output power:

$$P_u = P_{tr} - P_{jr} - P_{mec} = 5980.71 - 358.84 - 226.9 = \boxed{5394.96 \text{ W}}$$

Useful torque:

$$T_u = \frac{P_u}{2\pi N/60} = \frac{5394.96}{2\pi \cdot 1410/60} = \frac{5394.96}{147.58} \approx \boxed{36.55 \text{ N.m}}$$

c) Efficiency

$$\eta = \frac{P_u}{P_{in}} = \frac{5394.96}{6700} \approx \boxed{80 \%}$$

Exercise 03

Given Data

- Armature (induit): $U_a = 100 \text{ V}$, $I_a = 8 \text{ A}$, $R_a = 1.25 \Omega$, $N = 1500 \text{ rpm}$
- Field (inducteur): $U_{ex} = 200 \text{ V}$, $R_{ex} = 400 \Omega$ (flux Φ constant)

1. Nominal Back E.M.F. E

For a separately-excited DC motor

$$E = U_a - I_a R_a$$

$$E_n = 100 - 8 \times 1.25 = \boxed{90 \text{ V}}$$

2. Required Armature Voltage U'_a at $N' = 1000$ rpm

Since Φ is constant, $E \propto N$:

$$E' = E_n \frac{N'}{N} = 90 \times \frac{1000}{1500} = 60 \text{ V}$$

Then

$$U'_a = E' + I_a R_a = 60 + 8 \times 1.25 = \boxed{70 \text{ V}}$$

3. Joule Losses

Armature

$$P_{J,a} = I_a^2 R_a = 8^2 \times 1.25 = \boxed{80 \text{ W}}$$

Field

Current $I_{ex} = U_{ex}/R_{ex} = 200/400 = 0.5 \text{ A}$

$$P_{J,ex} = I_{ex}^2 R_{ex} = 0.5^2 \times 400 = \boxed{100 \text{ W}}$$

4. Electromagnetic Torque at $N = 1500$ rpm

Electrical power converted to mechanical:

$$P_{em} = E_n I_a = 90 \times 8 = 720 \text{ W}$$

Speed in rad/s: $\omega = 2\pi N/60 = 2\pi \times 1500/60 = 50\pi \text{ rad/s}$

$$T_e = \frac{P_{em}}{\omega} = \frac{720}{50\pi} = \boxed{4.58 \text{ N m}}$$